THE MATHEMATICS LEARNING EXPERIENCES OF FOUR IMMIGRANT STUDENTS

by

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Abstract

This study was designed to explore the mathematics learning experiences of immigrants students to Canada.

A thorough review of the related literature suggested that this exploration be guided by three categories: language, culture, and cognition.

To facilitate the exploration, grade 9 was chosen as the grade level and algebra as generalised arithmetic was chosen as the mathematics topic. Four research subjects were recruited, all of whom had been born and partially educated in some country other than Canada.

Each research subject went through two interviews. The first interview involved a series of questions based on the reviewed literature. The second interview specifically looked at algebra as generalised arithmetic, with the assistance of the Chelsea Diagnostic Mathematics Test for algebra. The data from the interviews were then unitized to allow for analysis.

The analysis challenged the view created from the literature that immigrant students' cultures, language and cognitive structures will affect their mathematics learning experiences. Factors such as a common mathematics curriculum 'culture' and social stability may be equally important. Mathematics teachers working with immigrant students should therefore be somewhat cautious when

applying views expressed in the ethnocultural literature to their classes.

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Table of Contents

List of Tables

CHAPTER ONE

Introduction

Purpose

Several years ago I had the pleasure of teaching mathematics in a secondary school for the indigenous inhabitants of Malaŵi, a small East African country. I returned to Canada with questions both about my ways of teaching and about my African students' ways of learning. Reading of the literature and my own experiences suggested that one's culture and language may have an effect on the ability to learn a specific type of mathematics - the type taught in Western eurocentric schools like those in Ontario and Malaŵi. If true, there would be serious implications for how mathematics is taught in increasingly culturally diverse classrooms. Could I find evidence of the role culture and language play in the learning of mathematics?

Rationale

To provide a rationale for the present study I first decided to find a definition of what mathematics is, and then address the manner in which it is taught and to whom it is taught in this part of southwestern Ontario. What I found is presented under four subheadings: western and other mathematics, mother tongue and mathematics, uniform treatment of mathematics students, and the immigrant situation in London.

Western and Other Mathematics

Formal mathematics, typical of what is taught in London schools, has its roots in ancient Greece and the Near East. It was adopted by the Europeans beginning in the twelfth century and for the next 800 years has been under the dominion of European or eurocentric mathematicians and scientists. The evidence of a eurocentric influence on the mathematics taught in Ontario schools can be seen when one looks at the prescribed topics in the Ontario Ministry of Education's Curriculum Guidelines (1985) and guides such as the London and Middlesex County Roman Catholic School Board's series of curriculum guides (1988), and then learns from historical sources where and by whom those topics were developed and refined (Bergamini, 1963; Hogben, 1968; Rogers, 1966). Those topics include algebra, formal geometry, graphs in the Cartesian plane, functions and relations, and differential calculus.

In contrast, scholars (Ascher, 1991; D'Ambrosio, 1985b) have identified an alternative mathematics, the mathematics used in city streets and rural villages around the world. In particular, D'Ambrosio (1990) defined what he calls ethnomathematics. This is the art or technique (*tics*) of explaining, understanding, or coping with (*mathema*) the sociocultural and natural (*ethno*) environment. D'Ambrosio saw ethnomathematics as a subject which "lies on the borderline between the history of mathematics and cultural anthropology" (1985a, p. 44), and noted that anthropologists' work which recognizes different modes of thought may lead to different forms of mathematics. As opposed to academic or formal mathematics, ethnomathematics is the mathematics practised by identifiable cultural groups. These groups can be as varied as tribal groups, labour groups, age groups or professional classes, all of which have their own "jargons, codes, myths, symbols, utopias, and ways of reasoning and inferring" (D'Ambrosio, 1985a, p. 46).

With respect to formal mathematics, D'Ambrosio saw the mathematics somewhat like expropriated cultural intellectual property. He wrote that, as well as containing "Platonic ciphering and arithmetic, mensuration and relations of planetary orbits [contributions of Galileo and Newton]" (1985a, p. 45), formal mathematics included

> the capabilities of classifying, ordering, inferring and modelling. This is a very broad range of human [activities] which, throughout history, have been expropriated by the scholarly establishment, formalized and codified and incorporated into what we call academic mathematics. But which remain alive in culturally identified groups and constitute routines in their practices. (1985a, p. 45)

In addition to that expropriation, D'Ambrosio added that academic or formal mathematics was the primal discipline driving the new society brought to the 'discovered' world by its European conquerors. As a result of this "civilizing mission" (D'Ambrosio, 1990, p. 20), a cultural genocide occurred.

Reacting to the new society, D'Ambrosio said that attempts to look at other mathematics are vitally important. He saw a benefit to building upon the natural, social and cultural roots of students, something he considered criminal to ignore (1990, p. 22). As far as the teaching of mathematics was concerned, D'Ambrosio believed that it should free people to be creative, to be able to pursue personal happiness and social fulfilment, and to achieve happiness (1990, p. 21).

Thus there is clearly more than one view of what mathematics is, and indeed more than one view of what kind of mathematics should be taught. Extrapolating from D'Ambrosio, it is reasonable to suggest that the formal mathematics taught in London schools may be seen by some peoples as the mathematics of oppression. It is possible that students arriving in Canada from other countries may have been exposed to, or were practiced in the use of, another form of mathematics. The mathematics used in our schools could be just as foreign - and possibly intimidating - to these students as their mathematics would be to us.

Mother Tongue and Mathematics

The work of Myers and Milne (1988) provides much insight into the complexity of determining what affects mathematics achievement. Myers and Milne were interested in the effects of language on the learning of mathematics in the United States. They engaged in quantitative research using the 1980 database built from the High School and Beyond survey of 28,000 senior high school students. Myers and Milne's research had two objectives. These objectives were to isolate the independent effects of language on achievement by controlling for other variables (such as socio-economic status), and to determine what intervening variables language works through to affect achievement. In their research Myers and Milne considered students' home language, primary language, reading and mathematics achievement, and other variables such as sex, family income, and home environment. They stated that English proficiency alone was not a sufficient predictor of mathematics achievement, suggesting that language proficiency should not be considered independently but rather in conjunction with some other factor or factors, yet to be identified.

Similar results were found by Secada (1992). In his survey of American research on race, ethnicity, social class, language, and achievement in mathematics, he cited several reports linking language background and English proficiency to mathematics achievement. Some of these reports were based on the High School and Beyond data; others included a California Department of Education report on 12th-grade mathematics achievement, and a report on tests given in both English and Spanish to bilingual Puerto Rican adults.

Secada discussed groups as socially created entities in which membership "is a negotiated and complex phenomenon that is not easily expressed, nor can it be easily described as a variable" (Secada, 1992, p. 640). This means it is inappropriate to say, for instance, that a student is German and therefore should excel at logic; it is a warning against group stereotypes. Secada determined that mathematics achievement should be treated as a social issue, and found powerful links between it and the factors of language and socio-economic status.

Hence, researchers have considered language as a factor affecting the learning of mathematics. Myers and Milne tried to account for home and primary language. Secada surveyed much of the literature on different language backgrounds, and found links to mathematics achievement. Given the attention that language has received from the research community, it might be important to begin exploring the relationship between a student's first language and its effect on his or her learning of mathematics.

Uniform Treatment of Mathematics Students

In London, Ontario, immigrant students are provided with English as a Second Language (ESL) classes in both publicly funded school systems and through community organizations. Examples of these ESL programs are the Northbrae Public School program described by Shawna Richer (1995, October 13) and the Fast Track program at the Portuguese Community Centre described by Norman De Bono (1995, October 11). But beyond language training there does not appear to be any activity that acknowledges the other factors which may affect mathematics performance. There are separate French and English mathematics programs, but within these there is no breakdown of treatment by culture or ethnic group. Regardless of their background, students receive the same basic program as their peers (the exceptions in London being the N'Amerind Friendship Centre's Sweetgrass School and the public board's few alternative programs).

The uniform treatment of mathematics students represents an assimilationist view of teaching and learning. That is, in order for one to be a successful mathematics student one must conform to a particular view of what mathematics is and how it is to be taught. This view appears to be in opposition to the Government of Canada's 1971 policy of multiculturalism (Masemann, 1991), commonly known as the "cultural mosaic."

Within the Ontario education system the cultural mosaic view is clearly expressed in Ministry of Education documents. For example, The Common Curriculum: Policies and Outcomes, Grades 1-9 (1995) states that

> The Common Curriculum is designed for all students; that is, it recognizes that programs must reflect the abilities, needs, interests, and learning styles of students of both genders and all racial, linguistic, ethnocultural groups (p.9).
>
> The curriculum must reflect the diversity of Canadian society . . . . Students are entitled to have their personal experiences and their racial and ethnocultural heritage valued, and to live in a society that upholds the rights of the individual (p.19).

Further, many teachers may not be aware of the different learning styles, and mathematical and cultural viewpoints of their ethnically and culturally diverse students. In a critique of the general lack of awareness, the National Council of Teachers of Mathematics (NCTM) stated in their Professional Standards for Teaching Mathematics (1991) that:

> Teachers should develop their perspectives on the nature of mathematics, the contributions of different cultures toward the development of mathematics, and

> the role of mathematics in culture and society (p. 132).
>
> Teachers should develop their knowledge of the influences of students' linguistic, ethnic, racial, and socioeconomic backgrounds and gender on learning mathematics (p. 144).

Statements such as these, which encourage teachers to be aware of, and value, the racial, linguistic and ethnocultural characteristics of their students, lead one to ask what those characteristics are and what their effects are on the learning of mathematics.

The Immigrant Situation in London, Ontario

In 1993 it was reported that London ranked sixth among all Canadian cities in terms of the proportion of the population made up of immigrants who arrived since 1988 (Population Studies Centre, 1993). The same study found that the number of immigrants arriving in London increased two and one half times when the intervals 1981-87 and 1988-91 are compared. The numbers were great enough for it to be seen reasonable to launch a multi-cultural newspaper, Global Village, in the city in 1995.

Unpublished data provided by Employment and Immigration Canada (1995) gave a breakdown of permanent residents whose intended destination was London for the years 1992 to 1994. As evidence of London's growing immigrant population and its impact on the school system, consider the data on those from Spanish-speaking Latin American countries displayed in Tables 1 and 2.

The 0-17 columns of Tables 1 and 2 show that there were approximately 233 (totals of the 0-17 groups for 1992, 1993, 1994) elementary and secondary school-aged children arriving in the city in the three years reported. Of these children, 163 had no proficiency in either English or French - the languages of instruction in London's schools - and 67 had no educational qualifications, which I understand to mean had not completed elementary school. These students would be obliged to enter the school system and, unless recognized as having the equivalent of two secondary school mathematics credits, would have to take mathematics.

The points detailed earlier suggesting a link between culture, language and the learning of mathematics would have been of little interest to teachers practising in London had it not been for the recent influx of new immigrants. Clearly the population of London is becoming more ethnically and culturally diverse, and therefore it is important to start asking how teachers should be working with those newly arrived students.

To conclude, it is important for educators in London, Ontario to be concerned about how culture, ethnic origin and language have an effect on the learning of mathematics. First of all, D'Ambrosio's work suggests that the

Table 1

Language Proficiency of Spanish-speaking Latin American Permanent Residents with Intended Destination London, Ontario

	1992		1993		1994	
Age Group	0-17	18+	0-17	18+	0-17	18+
Proficient in English	46	149	21	49	3	27
Proficient in French	0	0	0	0	0	0
Proficient in neither official language	76	99	70	79	17	25
Total	122	248	91	128	20	52

Table 2

Elementary and Secondary Educational Qualifications of Spanish-speaking Latin American Permanent Residents with Intended Destination London, Ontario

	1992		1993		1994	
Age Group	0-17	18+	0-17	18+	0-17	18+
No Educational Qualifications	35	10	22	4	10	3
With Secondary School Qualifications or less	87	154	69	75	10	29
With Higher Education	0	84	0	49	0	20
Total	122	248	91	128	20	52

mathematics of Ontario classrooms may be foreign, and even oppressive, to students from 'conquered' or 'discovered' countries. These students may have a totally different experience and understanding of what mathematics is, and be resistant towards 'western' mathematics. Thus culture has the potential of being an important factor in the learning of mathematics.

In addition, Myers and Milne, and Secada have presented quantitative data which suggests links between English language proficiency, mother tongue, and mathematics achievement. Statistically certain language groups appear to have a learning advantage, others a disadvantage. Language appears to have importance on par with culture for mathematics students.

Finally, mathematics educators in London need to be concerned about culture and ethnic origin if they are attempting to teach all students uniformly. Both Ontario government documents and NCTM standards point to the need for an awareness of language, cultural, and ethnomathematical factors on student learning. With London's growing immigrant population, teachers here need to consider alternative ways children learn in a culturally diverse classroom.

Exploratory Nature of the Study

What is not clear from the literature cited to this point is how culture and language affect the learning of mathematics. Myers and Milne's results suggested that language proficiency should be considered in conjunction with some other unspecified factors. Secada said that mathematics achievement should be treated as a social issue, but social issues are also complex. The NCTM Standards stated that teachers should develop their knowledge of the influences of students' linguistic, ethnic and racial backgrounds, but did not detail what the influences were.

The categories of culture and language have very broad and diverse bodies of literature. Further in-depth reading has allowed me to somewhat narrow down what could be affecting the mathematics learning experiences of immigrant students to several themes within language and culture. I also discovered that I needed to consider a third category: cognition. It is the importance of specific themes within these three categories to the mathematics learning experiences of immigrant students that are to be investigated with this research.

One way of examining mathematics learning experiences is to talk with students. Therefore, to help determine whether any of the themes within the categories of culture, language and cognition could be relevant to the learning of mathematics, the experiences of some immigrant students will be explored. Talking with students will allow me to examine the theories my readings identified in a different way - through the lived experiences of immigrant students. Exploring the theories with the students in this manner might help determine whether a particular theory merits more attention or ought to be discarded. In addition, I may also be able to identify unanticipated influences, and connections amongst what initially appeared to be isolated factors.

To facilitate this exploration one area of mathematics had to be selected. A review of the literature pertaining to this area of mathematics, algebra, as well as to language, culture and cognition, follows in Chapter Two.

CHAPTER TWO

Review of the Related Literature: Language, Learning and Culture

The intent of this study is to investigate how culture and language affect the learning of mathematics. However, trying to understand how a student knows a concept or idea is not a simple task. There are many inter-related and intertwined factors at play, which undoubtedly vary from individual to individual. Initially, as a result of reading and academic course work, I had envisioned that those factors could be cleanly distinguished using the two headings of language and culture. However, after further reading, I realized that cognition could not be ignored as a factor, nor easily isolated without considering language or culture. Within our minds language and culture affect us, influenced it seems by cognition. Because of that observation, this chapter is organized under the three headings of language, culture and cognition. However, the boundaries among these are fuzzy, and often the distinctions are arbitrary.

Language

My feelings or suspicions of how students from other countries or cultures learned mathematics became much more focussed when I read a paper by Berry (1985). Berry wrote of the experiences of mathematics students in Botswana, a country in the same general area of southern Africa as Malaŵi. His descriptions both of how the students were taught, and of how the students responded to the teaching, struck a common chord with my own experiences in Malaŵi. The reasons Berry gave for the students' responses effectively launched my own research.

Berry described the experiences of two 'typical' Botswanan students, one elementary and one secondary. The elementary student, Mothibi, had lived all his life in a Botswana village. He was taught in Setswana, using a carefully translated British curriculum. As the school year progressed, Mothibi's teacher found that Mothibi was not reacting to the mathematics curriculum as the teacher's manual predicted, and Mothibi was starting to simply memorize material.

The secondary student, Lefa, was studying in English, and his textbook was an Africanized edition of a British book. Despite the text and teacher's manual being structured to encourage independent thinking and discovery, Lefa only seemed to learn slowly and by rote memorization.

Berry conducted a literature survey to attempt to find an explanation for his observations. From the survey he believed that the key to the students' difficulties lay in the fact that their materials were derived from those of a different cultural and language group, namely the British. Berry referred to the Sapir-Whorf hypothesis, stating that the structure of a person's language had an influence on his or her cognitive processes. Thus, for example, the student whose ways of knowing were formed by immersion in an oral-based culture would have difficulty with materials whose authors' ways of knowing were formed by immersion in a writing-based culture.

Berry's conclusions gave me some insights into the questions derived from my Malawian experience. However, in order to explore the situation for immigrant students in Ontario, I needed to investigate further. Berry's work suggested an intimate link between language and cognition. Before proceeding in that direction, however, I first needed to search for other language implications relating to mathematics for students working in a second language. Discussion of three of these themes now follows: language proficiency, bilingualism, the words or symbols used by a language.

Proficiency in Working Language

Spanos, Rhodes, Dale, and Crandall (1988) studied students learning to function in English as their second language. They collected data at three U.S. colleges by analyzing algebra texts and tests, by recording students' discussions as they worked on math problems, and by interviews within small groups with forty-six algebra students. Some of the students interviewed were Hispanic, some had limited English proficiency, and some were native

English speakers. Spanos et al determined that there is a minimum level of English proficiency required for a student to function effectively in cognitively demanding tasks, and that this level can take five to seven years to develop in a student's second language.

In addition, Spanos et al spoke of CAMP - Cognitive Academic Mathematics Proficiency - which is the cognitive knowledge of mathematics embedded in a language which is structured to express that knowledge. Spanos et al called the language part of this the mathematics register, and suggested that a language approach to mathematics education is required in order to develop this register. For example, a word problem such as

> Find a number such that seven less than the number is equal to twice the number minus 23

will not be understood by minority students lacking relevant language skills, even though they might understand the individual words per se. Spanos et al then went on to describe problems specifically related to syntactics and semantics.

Thus it might be argued that a student arriving in an Ontario classroom from a non-English speaking country needs to attain not only proficiency in English but also in the language of the mathematics classroom in order to be successful. Admittedly native English speakers also have to learn the language of the mathematics classroom.

Nevertheless, depending on when the students arrive in Canada, they could find themselves behind their native English-speaking peers who have already been exposed to some of the language of the mathematics classroom.

Bilingualism

Another relevant language theme that arose from my readings was on the effects of bilingualism. A number of scholars attempted to identify characteristics specific to students who could comfortably function in two languages. One such scholar was Saxe (1988). Saxe conducted an analysis of studies that related language background to mathematics achievement. He distinguished his findings as pertaining to either direct effects or indirect effects.

Writing about direct or intrinsic effects of language on mathematics achievement, Saxe determined that it was reasonable to suspect a relation between the process of acquiring two languages and a student's understanding of certain mathematical concepts. The bilingual students in the studies he analyzed more frequently demonstrated an understanding of number symbol properties than their monolingual peers. He concluded that growing up bilingual does positively influence some aspects of a child's cognition about the arbitrariness of numerical conventions.

Another scholar who explored the effects of bilingualism is Clarkson (1992). For his starting point he referred to the theories of Cummins, who believed that bilingual students developed deep cognitive structures not solely dependent on language. Because of these structures, instruction in one language could develop skills in the other language, and underlying conceptual and linguistic proficiency would be enhanced.

Clarkson conducted his research in Papua New Guinea. He undertook a comparative study involving a group of 232 bilingual students, whose original language was Melanesian Pidgin but who used English in school, and 69 monolingual students from international schools. Clarkson's results indicated that students competent in both English and their mother tongue had an advantage in mathematics achievement over students competent in just one language. This was despite the fact that the monolingual group in the study had significant material advantages.

An implication of Saxe's and Clarkson's work for an immigrant student in Ontario is that bilingualism could be a positive factor in learning mathematics. In addition, the authors' works were examples of research literature that suggested to me the importance of cognitive structure and the links between cognition and language.

Number and Concept Words or Symbols Used in Particular Languages

A third example of the role language could play in the learning experiences of immigrant students was with the words used by a language to express numbers or mathematical concepts. These words can present difficulties with learning mathematics in English, as was demonstrated by Denny (1980). He investigated Inuktitut, the language of Canada's Inuit, and found that some word meanings did not have a direct correlation to English and others had much richer meaning. For example, in some of the Inuktitut dialects the number seven is "marruungnik arviniliit" (Denny, p.199), but the first word is like their word for two (marruuk) and the second is like the word for six (arviniliit). As Denny pointed out, in eurocentric thought how can one get seven from two and six? It turns out that the numbers 6,7, and 8 are "those on the inside of the right hand" when you count on your fingers. Consequently the word for 7 means "the second one in the group 6,7 and 8" (Denny, p.201). An Inuktitut word for circle is something like "angmaluqtuq" (Denny, p.200), which does not just mean 'it is circular' but can also refer to irregular shapes like pebbles and ponds. For a final example, Inuktitut vocabulary for spatial location is much richer than in English - due perhaps to the way the Inuit have of giving directions in the relatively featureless Arctic - so phrases like 'start your line here and end there' fail to tap into the potential of their language.

Further examples of peoples or groups whose linguistic or cultural features could handicap or benefit their mathematics learning experiences were given by Saxe (1988) and Ascher (1991). Saxe contrasted the counting system of the Oksapmin people of Papua New Guinea, which was based on twenty-seven body parts, with the regular and systematic number words of the Japanese. According to Ascher (1991), the Nahuatl of Central Mexico base their number system on their words for 1, 2, 3, 4, 5, 10, 15, 20, 400, 800 and on a cycle of twenty, and the Yuki of California use cycles of eight because that is the number of spaces between the fingers of both hands. Ascher also reported that in the Dioi language of southern China there are fifty-five different number classifiers, and it is considered insufficient and improper to use a number without a classifier. It is quite possible that an immigrant child exposed to one of these cultural or linguistic phenomena will arrive as a student in a Canadian classroom.

Given these examples, it seemed reasonable to agree with Saxe's conclusion that "[s]tudying cultural supports for mathematics development and how children utilize different cultural backgrounds in coping with school mathematics curricula can offer insights about the sources of language minority children's successes and failures in the mathematics classroom" (1988, pp.61-62).

Language is the obvious starting point when investigating the mathematics learning experiences of immigrant students, for if they cannot speak the language of the classroom they will not be able to participate in the learning process. Thus it was not surprising that the reviewed literature provided language proficiency as an important theme. What was unexpected was the suggestion that a student's mother tongue, or the level of the student's bilingualism, could positively affect his or her learning of mathematics. These two latter points are clearly worthy of further study.

Culture

The next major area of my reading focussed on cultural concerns. The word 'culture' can be used to talk about the way the citizens of another country live and act, to talk about the ways people in one's own society live and act, or even how certain occupational groups within one's own city behave. And culture varies, for as Ascher (1994) wrote:

> There is no question that no culture has remained pure or unchanged. The students are more diverse and many students are part this and part that - a wonderful, beautiful mixture. The more we ignore the variety of cultures, the less the students will understand what we are talking about. (p.41)

Ascher added that "[a]t all levels of schooling, the teaching of subject matter is intertwined with the teaching of culture" (p.43). Four particular aspects of culture relevant to the teaching of mathematics follow: culturally different curriculum, multiple systems within a culture, cultural inhibitors, majority versus minority culture.

<u>Culturally Different Curriculum</u>

As well as considering the connection between language and cognition, Berry (1985) defined two types of problems with using a culturally different curriculum. Type A problems occur when the language of instruction is not the student's mother tongue. The Botswanan students in Berry's research were familiar with the mathematical symbols and operations, but had problems understanding the English words in the exercises. They were not working in their mother tongues, and so can be considered Type A.

However, Berry argued that their difficulties were more than simply with translation, and he had another category to accommodate this - Type B problems. These problems are due to the distance between the cognitive structures of the student and those of the instructor or materials. To illustrate this, Berry reported the statement of a Botswanan university student who claimed to do all mathematical thinking in English: "This makes mathematics very difficult, since I do not think easily in English, but mathematical proofs cannot be done in Setswana" (1985, p. 21).

Hence, it is interesting to speculate whether or not immigrant students find the Ontario mathematics curriculum culturally different, and therefore experience Type B problems. For example, the existence of Type B problems would raise the challenging question of how a teacher from a written tradition like Ontario's should attempt to teach a student from an oral tradition.

Multiple Systems Within a Single Culture

Whether or not there are Type B students within our own culture, we still have multiple ways of expressing certain concepts. For example, in a recent discussion in my grade 9 math class, the students were asked to determine an individual's distance from work. In the formal sense one would expect the answer to be expressed in kilometres or some other unit of linear measure, but many students gave an answer like, 'He is 20 minutes from work.' Minutes are of course units of time, not distance, but are also informally and commonly used for expressing distance. D'Ambrosio (1985b) discussed similar examples with children practicing "applicable" (1985b, p.81) mathematics before entering school and being exposed to the school's mathematical practices.

Instead of distinguishing distinct non-intersecting types of mathematics such as 'street mathematics' and 'school mathematics', Nunes (1992) investigated how multiple mathematical systems can exist within a single culture. Based upon an extensive review of relevant literature, she described written practices as being rule-based, and focussed on what she described as cultural amplifiers - the resources in a culture that allow for an increase in a person's ability to function in certain areas.

Within mathematics, Nunes identified several amplifiers. For example, numeration systems amplify the person's capacity to count and register numbers beyond the limits of his or her memory. Place value systems increase the person's range of counting and calculation beyond what is possible orally or by manipulating objects. Nunes said these systems vary across cultures and even within a single culture. For example, the generation of students just entering secondary school in London have had no exposure to the Imperial system of measurement and have no idea how many inches there are in a foot.

In a vein similar to Nunes, Presmeg (1988) believed it possible for some people to successfully live with the influences and systems of several cultures. For those people, their own particular culture would be a sort of gathering point for the other cultures. She was writing in South Africa during the time of apartheid, and found that some students could successfully come to terms with the either-or conflict between eurocentric and traditional values. Her best known model for this is the Zulu Chief Mangosuthu Gatsha Buthelezi, who seemed equally at home in leopard skins and tassels while visiting a kraal or in a business suit negotiating with the white government.

<u>Cultural Inhibitors</u>

Leap (1988) had a long involvement with the Ute

Indians of northeastern Utah. To study their mathematics achievement, he first compared mathematics test results of Ute and non-native elementary students and then interviewed eighteen of the Ute students termed low achievers by their teachers and school authorities. Leap identified three culture-based factors that were associated with poor performance of the Ute Indians: home-school disagreements on the role of parents and/or tribal members, Ute students' acceptance of traditional Ute commitments to personal self-reliance, differences between text and/or curriculum and Ute counting and grouping. These he found while investigating the reasons for Ute students' avoidance of and poor performance in school mathematics classes. Finding these reasons was important as they were restricting Ute independence; by avoiding math and math-intensive courses the Ute were restricting their career choices to those which favoured qualitative skills. Consequently there were virtually no Ute scientists, engineers, and so on, thus forcing a dependence on non-Indians.

For Ontario First Nations' students, Donna Young (personal communication, October 25, 1994), Director for the London District Native Education Agency Project, identified some factors that could directly or indirectly affect the learning of mathematics. These factors were a lack of linear concepts, variances in learning styles and in concepts of family and community, the relationship to the natural environment, and linguistic differences. Because some of the First Nations languages do not accept mathematical terms easily, she reported that some Native immersion schools teach mathematics in English.

Ascher (1994) provided another example of a cultural characteristic which could inhibit a student's learning in an Ontario mathematics class. A member of the Kpelle people was given the following problem:

> All Kpelle men are rice farmers. Mr. Smith is not a rice farmer. Is Mr. Smith a Kpelle?

The person would not answer the question. One might conclude that he or she did not understand the question, but within the Kpelle culture if one did not know an individual one could not answer questions about that individual. Hence, as the Kpelle being questioned did not know Mr. Smith, he or she could not answer because of the conflict with the Kpelle value system.

Majority versus Minority Culture

An additional perspective on culture comes from Ogbu's (1992) study of minority groups. He reviewed research literature to investigate the relationships which exist in the United States between minority cultures and the mainstream culture. He realized that some minorities do succeed in school despite their different language and cultural backgrounds, and sought a reason for this fact.

Ogbu concluded that the key factor is the strength of those minorities' desire to assimilate and/or become successful. He defined two categories and their associated characteristics: voluntary minorities with primary cultural differences, and involuntary minorities with secondary cultural differences.

Voluntary minorities are those who willingly enter a society. Ogbu called the differences between their own culture and that of the society they entered primary. These differences are overt, including their way of dressing and their religion. While perhaps providing fodder for teasing and making the school experience less pleasant, these differences ultimately do not hinder the voluntary minority member's ability to eventually be successful in school.

Involuntary minorities are those such as the American Native Peoples and the black slaves - those who are unwillingly brought into a society. Ogbu called their cultural differences secondary, to distinguish them from the primary cultural differences. Secondary differences develop after contact with the dominant group, and involve things like different communication styles and interaction styles, and cultural inversion. This last item means valuing what is directly opposed to the dominant culture. The characteristics of involuntary minorities remain long after the events which instigated them - such as being conquered or enslaved - have faded into history.

Reading in the area of culture has given me cause to reflect further on my Malawian experiences. Certainly many of the theories pertain to attempting to implement a westernized curriculum in a non-western country. What remains unanswered are several interesting questions relating to students immigrating to Ontario. Do immigrant students notice a cultural bias in the Ontario curriculum? Will an immigrant student perceive or practise more than one mathematics? Are some immigrant students constrained from fully participating in an Ontario mathematics class by a feature of their culture? Or are there some constraints imposed by being a member of an ethnic group which suffered the effects of being a minority culture? Based on the reviewed literature, it seems plausible that the answers to these questions are crucial when investigating the mathematics learning experiences of immigrant students.

Cognition

After embarking on my search of the research literature pertaining to how culture and language affected the learning of mathematics, I soon realized that there was a third factor I would have to consider. This factor provided a way of linking language and culture, sometimes even blurring the distinctions between them. It kept appearing either explicitly within the literature or implicitly as I reflected upon the implications of the literature, suggesting that some of the authors could not talk about culture or language in isolation from this third factor. The factor is cognition.

The Concise Dictionary of Psychology (1990) defines cognition as a "general term which includes all the psychological processes by which people become aware of, and gain knowledge about, the world" (p. 26). The Concise Encyclopedia of Psychology (1987) says that cognition "comprises all mental activity or states involved in knowing and the mind's functioning, and includes perception, attention, memory, imagery, language functions, developmental processes, problem solving, and the area of artificial intelligence" (p. 202). Lefrançois (1991) says that "cognition is knowing" (p. 59).

Although I reviewed the works of many authors who wrote about the cognition construct, it was Vygotsky's (1934/1986) book Thought and Language that most closely spoke to my experiences in MalaŴi. He recognized that our thoughts and the ways we recognize and express or vocalize those thoughts are tied together in a complex way. Language is more sophisticated than merely speaking. In fact, "[t]hought and speech turn out to be the key to the nature of human consciousness" (Vygotsky, 1934/1986, p.256).

Vygotsky conducted his research on cognition in Russia during the early decades of this century. Through his research he developed a theory of the communication process, which has relevance for my exploration of language and culture. It can be simplistically illustrated in the following manner:

MOTIVE → THOUGHT → INNER SPEECH → SEMANTICS → PHONETICS.

The motivation for one's thoughts can be provided by one's culture. Thought involves cognition. Inner speech, semantics and phonetics involve word meaning and language to express the motivated thoughts. What one actually says can provoke a response which motivates another thought. Thus Vygotsky has created a theory of cognition which incorporates language and culture, and implies that they have an important role in the development of one's cognitive structures.

The relationship between thought and language is thus well worth exploring, especially when one recognizes that both language and thought are required for the learning of mathematics. In addition, if language and thought contribute to cognitive development, it is interesting to speculate on whether immigrant students have formed cognitive structures which affect their mathematics learning experiences. However, the relationship between thought and language still remains a complex construct. Vygotsky himself believed that word meaning was the key to investigating the construct. Other researchers have identified additional aspects of Vygotsky's theory which pertain to the learning of mathematics (see, for example, Sierpinska, Taylor, Teslow, Wilson, and others in Focus on Learning Problems in Mathematics, 15(2&3), an issue devoted entirely to Vygotsky's work). From my readings four themes arose, which I used to organize this part of the literature review. The themes are word meaning, level of understanding, motivation and attitude, and concepts.

Word Meaning

Vygotsky (1934/1986) stated that, before he began his research, thought and language had been examined in isolation. He determined that this was an erroneous approach, likening it to studying hydrogen and oxygen to determine the nature of water. Vygotsky described thought as a cloud shedding a shower of words. The words are thoughts realizing themselves, but just as clouds do not always mean rain will fall so too do thoughts sometimes remain unrealized. Because Vygotsky saw thought and word or language as interrelated, he believed there should be a way to analyze the complete entity. This was the genesis of his research.

Vygotsky and his team of researchers decided to try using word meaning as the unit of analysis. Word meaning was considered a 'unit' as it could not be further reduced and still retain the properties they wanted, which was uniting thought and speech into verbal thought and thus retaining the properties of the whole of the communication process. They called their technique "semantic analysis - the study of the development, the functioning, and the structure of this unit, which contains thought and speech interrelated" (Vygotsky, 1934/1986, p. 6). One conclusion from their research was that word meaning was the appropriate unit for the study of the development of verbal thought. Going further back and looking at word without meaning resulted, in their opinion, in dealing with an empty sound. As a result of their research, Vygotsky determined that thoughts are mediated internally by word meanings and externally by signs, the latter linking with the importance of motivation alluded to earlier.

Anshen, in her preface to Chomsky's (1993) book Language and Thought, expressed views supporting those of Vygotsky. These views are a poetic contribution towards my suspicion that the meanings assigned to words in a particular language influence the way a native speaker of that language develops his or her cognitive structure. Anshen wrote that

> [t]hought itself must be accompanied by a critical understanding of the relation of linguistic expression to the deeper and most persistent intuition of man. It is by virtue of the provocative power of language which grasps, shakes, and transforms that human beings become human. For nothing really human can be so without this meaning, whether the language be uttered

> or silent. (p.10)

Anshen concluded that

> [l]anguage thus becomes indispensable not only for the construction of the world of thought but also for the construction of the world of perception, both of which constitute the ultimate nexus of an intelligible communion, spiritual and moral, between us all. Language is an energy, and activity, not only of communication and self-expression but of orientation in the universe. (pp.11-12)

Skemp (1978) addressed a different aspect of word meaning. He cautioned that word meaning could be a "faux ami" (p.20) as far as understanding in mathematics was concerned. The *faux amis* are words which have the same spelling and pronunciation, or very similar spelling and pronunciation, but have different meanings. A simple non-mathematical example of this was of an Englishman in the United States ordering what he called a 'biscuit' and being given what he would call a 'scone'; he should really have ordered a 'cookie'. The 'biscuit' is an example of the same word used in the same language and context, with two meanings whose differences are basic but non-trivial. Skemp observed that not all occurrences of this phenomenon will be as harmless as the biscuit; one should expect instances of serious confusion.

In the field of mathematics, Skemp cited the word 'understanding' as being a faux ami. When a mathematics teacher asks a student, "Do you understand?" and the student responds, "Yes, I understand" the teacher and student could be referring to two very different forms of cognition; namely, relational and instrumental.

Thus word meaning would appear to be the crucial variable when studying how language, culture and thought affect the learning of mathematics. With respect to my research, word meaning is particularly important for two reasons. First, if Vygotsky was correct about word meaning being the link between thought and language, then this implies that immigrant students may have their meanings - and hence their cognitive structures - formed by their first language. These cognitive structures could be significantly different from those developed by the English language, and consequently could have presented obstacles both to the students as they began to learn in an English-speaking classroom and to their teachers who may not have known how to best present material in order to speak to those cognitive structures.

Second, if Skemp is correct about the faux ami, the meanings assigned to words by immigrant students as they made the transition to learning in English may not have been what their teachers expected or wanted those meanings to be. These differences could have led to difficulties in learning mathematics.

Level of Understanding

Through his analysis of word meaning, Vygotsky realized that word meanings did not remain static, they evolved - strengthening, weakening, changing. As the meaning changed so too did the relation of thought to word. From these observations he determined that there were stages in the development of word meaning, stages dependent upon the "continual movement back and forth from thought to word and from word to thought" (Vygotsky, 1934/1986, p.218). This is similar to what Spanos, Rhodes, Dale, and Crandall (1988) concluded in the Language section, which was that students moved through levels as they learned to function in English as a second language.

Skemp's (1978) approach to the theme of Level of Understanding was different. Instead of writing about stages or levels of understanding, he addressed different *ways* of understanding. He discussed one aspect of cognition relevant to the teaching of mathematics, specifically how a student understands a concept. After identifying 'understanding' as a faux ami, Skemp then noted that there are two kinds of understanding, relational and instrumental. Relational understanding means "knowing both what to do and why" (Skemp, p.20). Instrumental understanding means being able to use rules without reasons, and is exemplified by the student who wants to know how to obtain the correct answer without knowledge of why the method works. Skemp said that relational understanding, though requiring more work to obtain, was the more important way of understanding for mathematics students.

Motivation and Attitude

Vygotsky also determined that thoughts were mediated externally by signs, and internally by word meanings. The motivation for the thoughts included one's desires, needs, interests, and emotions. For Vygotsky it was only possible to understand an individual's thoughts if one understood the thoughts' affective-volitional basis.

Working from Vygotsky's ideas, Wilson, Teslow and Taylor (1993) identified three motivation-related contributions to the theory of teaching mathematics. The first is the primacy of the social factor, as it is key to providing the motives for the development of thoughts. Second, and somewhat related, is the importance of the connection between motivation and attitude development. Third, and again somewhat connected to the first, is the role of dialogue. It is worth noting that dialogue also contributes to the development of word meaning.

While she was investigating the effect of a society's culture on teaching practice, specifically regarding attitudes, Taylor (1993) wrote on how attitudes affected cognition. Her work was also derived from Vygotsky's writings, especially his zone of proximal development which had to do with the distance between an individual's current development level and his or her potential level. She discussed how signs and symbols influence individual development, in other words how culture is internalized. From there she looked at how attitudes (for example, towards mathematics class) are formed, and not surprisingly said they are dependent upon culture, family, socialization, schooling experiences, relationships with role models and mentors, and so on. Awareness of attitudes is important because the "intellectual development of many students is limited by educators' failure to recognize the importance of interactions as social processes affecting learning" (p.13).

Thus the literature says that attitude and motivation are connected to cognition, and suggests a link between them and culture. In addition, it may also be worth remembering here D'Ambrosio's opinions on civilising missions and cultural genocide, and the attitudes these activities may have created amongst some indigenous peoples.

<u>Concepts</u>

The final extension of Vygotsky's work which became appropriate for my investigation into culture and language has to do with concepts. Sierpinska (1993) used Vygotsky's theory of thought and language, plus her own observations of ten- to fourteen-year-old school children, to examine how concepts develop. She studied this development as it related to a student's learning of mathematical concepts, and determined that it required the learner to have knowledge of his or her own thinking processes. These processes included generalization, identification of features of objects, comparison and differentiation, and synthesis. It is quite likely that a student from a different culture will have different ways of performing these processes; recall for example that Denny (1980) found the Inuktitut word for circle not only means 'it is circular' but can also refer to irregular shapes like pebbles and ponds. Somewhat related to the ideas in the Level of Understanding section above, Sierpinska said that it was impossible for an adult to simply transmit his or her way of thinking to a child; the child has to work at a child's level. This would require knowing what level or stage the child was at.

The reviewed literature on cognition has added an additional layer of interest to several questions regarding an immigrant student's experience of learning mathematics. Will an immigrant student's way of understanding mathematics correspond to what is expected of him or her in an Ontario classroom? Is a student's level of understanding affected in any way if he or she began learning mathematics in another country and language? Does a student's culture or ethnic origin affect his or her motivation, or the way he or she develops concepts? Perhaps these questions could initially be associated with a culture or a language, but I now see them as asking about a student's cognitive structure - and how that structure developed.

Before proceeding to the description of my research methodology, the mathematics topic used in my research needs to be reviewed. As was noted in Chapter One, one topic in mathematics was needed in order to effectively address the exploration of the three factors. The following section looks at the literature relevant to that mathematics topic: algebra.

Review of the Related Literature: Mathematics

For this study, the area of mathematics known as algebra as generalized arithmetic has been chosen as the avenue with which to explore the learned mathematics of the immigrant students. I chose algebra in general because it is a fairly new topic to secondary school students; consequently I hoped for an opportunity to investigate how a student's new concepts were developing. In addition, algebra requires the abstraction of the arithmetic rules students have used from their earliest mathematics lessons, so I also saw an opening for discussion with immigrant students on how their understandings of existing concepts were transforming. What follows is a review of the literature that led me to narrow my choice of topic to algebra as generalized arithmetic.

Algebra as Generalised Arithmetic

There are several interpretations of algebra. One interpretation is that of generalized arithmetic (Usiskin, 1988). Generalised arithmetic involves the use and meaning of letters or variables as pattern generalizers. Related tasks include substitution, simplifying expressions, and constructing, interpreting and solving equations. For example, 3 + 5.7 = 5.7 + 3 can be generalised as a+b=b+a. Slightly more complicated is taking the arithmetic values from various almanacs for the world record for running the mile and deriving the following equation for the world record time in seconds for running the mile Y years after 1900:

$$T=-0.4Y+1020$$ (Usiskin, p.11).

Usiskin believed that generalised arithmetic has historical importance for its role in the development of formal mathematics. He pointed this out by noting that within fifty years of generalised arithmetic's introduction to Western mathematics the field of analytic geometry was invented, and within one hundred years calculus was invented. He also challenged disbelieving readers to try to describe the rule for multiplying fractions without using algebra (as generalised arithmetic) as a further illustration of its importance.

Use of Variables

Küchemann (1981) showed that, because letters ("variables") can be used to represent a single value, a group of values, or even a mathematical concept (such as using A for area), and because there is a move from the concrete (use of numbers) to the abstract (use of letters), many students had difficulty with generalised arithmetic. The results of an algebra test, devised to sample a wide range of generalised arithmetic activities such as substituting and simplifying, and constructing, interpreting and solving equations, suggested that "the extent to which the letters are meaningful to children will be of vital importance in determining item difficulty. Furthermore, it seems likely that children may give *different meanings* to the letters, which in turn would affect item difficulty in that some items might be solved in unexpected ways" (p.103). It is because of Küchemann's reference to meaning, in conjunction with the previously discussed literature relating meaning to language and culture, that made this aspect of algebra attractive for my planned research.

Küchemann identified six categories used by his students when interpreting letters. They were:

- letter evaluated - the letter is assigned a numerical value from the outset of the student's answer

(for example, if $a+5 = 8$ then $a = 3$)

- letter not used - the letter is ignored, or acknowledged but not given a meaning (for example, if $a+b = 43$ then $a+b+2 = 45$)
- letter used as an object - the letter is treated as representing an object, or as an object itself (for example, $2a+5a = 7a$)
- letter used as a specific unknown - the letter is treated as a specific but unknown number which can be directly operated upon (for example, add 4 onto $n+5 = n+9$)
- letter used as a generalised number - the letter is treated as being able to take on several values (for example, if $c+d = 10$ and c is less than d then $c < 5$)
- letter used as a variable - the letter is treated as representing a range of unknown values, and a systematic relationship exists between two of these ranges (for example, 2n is larger than n+2 when $n > 2$).

He said that students who only made use of the first three categories did not really grasp the real meaning of the beginnings of algebra; that is, they had no notion of generalized arithmetic.

CSMS Levels of Mathematical Understanding

Küchemann conducted his research into use of variables as a member of the Concepts in Secondary Mathematics and Science (CSMS) research team at Chelsea College, University of London. This team developed the Chelsea Diagnostic Mathematics Tests. These tests were used as diagnostic instruments to determine a student's level of understanding in a number of mathematical areas, including algebra as generalised arithmetic. They were administered to approximately ten thousand English school children between the ages of 13 and 15 in the late 1970s.

The algebra portion of the Chelsea Diagnostic Mathematics Tests had correlations ranging from 0.6 to above 0.7 with the other Chelsea Tests. In addition, it was found to have a correlation of about 0.7 with raw scores on the NFER Non-Verbal Test DH. Test DH, also known as the Calvert DH Test, was a test of non-verbal reasoning used in England at the time the Chelsea Diagnostic Tests were being developed (Hart, Brown, Kerslake, Küchemann, & Ruddock, 1985; Küchemann, 1981).

From the algebra portion of the Chelsea Diagnostic Tests, Küchemann (1981) identified four levels of understanding for generalised arithmetic. Students at Level 1 could deal with problems that were purely numerical or that could be successfully solved using the first three categories described above, and for comparison's sake corresponded roughly with a Piagetian early concrete sub-stage. Level 2 was a more complex version of Level 1 and contained some ambiguity. Students at Level 2 still could not consistently cope with the last three categories. In Piagetian terms this corresponded to the late concrete sub-stage. The next level, Level 3 (Piagetian early formal sub-stage), was for students who could use letters as specific unknowns, but only to a simple degree. Finally, Level 4 was for those who could function with all of the last three categories described above, and corresponded with a Piagetian late formal sub-stage.

Küchemann reported the levels of understanding as displayed in Table 3. Notice that the percentage of Level 0 and Level 1 students dropped dramatically from 60% with the 13 year olds to 41% with the 14 year olds. Küchemann speculated that the performance difference noted between the 13 and 14 year olds could be due to an increased familiarity with and emphasis on algebra. In Vygotskian terms this increased familiarity could be due to the ongoing movement between thoughts and words related to algebra, mediated by developing word meaning, and resulting in better developed concepts.

Küchemann also noted that in each age group over fifty percent of the students were not able to cope with items that could be properly called algebra. The age groups he studied correspond roughly with the age range for Ontario's Intermediate school division, which is also the division in which algebra is introduced. Given the focus of the

Table 3

Percentage of Students at each Algebra Level (Hart, 1981, p.116)

Age	13 years	14 years	15 years
Level 0	10	6	5
Level 1	50	35	30
Level 2	23	24	23
Level 3	15	29	31
Level 4	2	6	9

present research on the early secondary school years, algebra as generalised arithmetic is thus a reasonable topic to choose. Further, the Chelsea Diagnostic Test is available as a well-defined diagnostic tool which allows for some speculation and discussion on cognition.

Summary

This chapter has focussed on literature related to the learning of mathematics as it is affected by language, culture and cognition. I began considering only language and culture, but cognition has emerged as an additional critical factor. Numerous themes relating to these three factors resulted, themes which could guide my investigation into an immigrant student's learning of mathematics. Cognition themes which were discussed illustrated how thought and language could work together through word meaning, how understanding could move through stages or levels, how motivation and attitude could contribute to cognitive development, and how concepts could be developed. In addition, these cognition themes overlap. Overall it seems quite plausible that an immigrant student's cognitive structures could have been developed by his or her language or culture, and therefore it will be interesting to examine the effects of those structures with some immigrant students.

Also important to the learning of mathematics are language issues such as proficiency, the effects of bilingualism, and the consequences of the words or symbols used by a language. Although I have already stated that language seems to play a part in cognition, these three language themes appear to be worth examining separately from cognition. The same is true for the themes I have identified pertaining to culture. Cultural issues important to the learning of mathematics include that the curriculum could be culturally distant for a student, or that the student's culture could contain either different ways of addressing mathematics or characteristics which inhibit learning. There could also be effects from being a member of a minority culture.

The reviewed literature has suggested all of these themes are important for the learning of mathematics, with particular implications for immigrant students in Canadian classrooms. As a result, I began to sketch out a plan that would allow me to explore these themes with some immigrant students. It would not be a straightforward process but, using the methods employed by some of the reviewed researchers as a guide, a methodology which is now detailed in Chapter Three was developed.

CHAPTER THREE

Methodology

Chapter Three begins by describing the choice of mathematics topic used for my research. A rationale for the research design and introduction to the four students who agreed to participate in the research follows. The method of data collection is then described, divided into a first and second interview. Finally, the method for analysis is discussed.

Mathematics Topic

In Ontario, mathematics is mandatory to the end of grade 8 and a student needs two secondary school mathematics credits in order to successfully complete secondary school. In other words all Ontario secondary school students must take at least grade 9 mathematics. Thus, because of its explicit inclusion in the mathematics curriculum, every student in grade 7 to 9 must take algebra. A good grounding in algebra at this level is also crucial for success in future mathematics courses because, regardless of whether the student continues in the general or advanced stream, he or she will be exposed to algebra again.

In addition, algebra is relatively new to all students at the grade 7 to 9 level. This makes it appropriate as a topic to use as an investigative tool because the ideas will still be relatively fresh. Students will have been exposed to other mathematical topics for several years and consequently because of familiarity may not be able to identify how or why they perform certain mental tasks. The newness of algebra should make it easier for a researcher to identify factors affecting students' learning.

But algebra includes many topics even at that level, thus a specific subset of the algebra curriculum - the use and interpretation of letters in generalised arithmetic - was selected as a focus. As described in Chapter Two. generalised arithmetic includes tasks such as substitution, simplifying expressions, and constructing, interpreting and solving equations.

Research Design

A qualitative research design was chosen because of the type of data I wished to obtain. As defined by Bogdan and Biklen (1982), qualitative research has the following five features:

1. Qualitative research has the natural setting as the direct source of data and the researcher is the key instrument.
2. Qualitative research is descriptive.
3. Qualitative researchers are concerned with process rather than simply with outcomes or products.
4. Qualitative researchers tend to analyze their data inductively.
5. "Meaning" is of essential concern to the

qualitative approach. (pp. 27-29)

In particular it was the second, third, and fifth features that determined the choice of a qualitative approach. Being descriptive meant that the participating students would be known as individuals. An exploratory process was necessary because this research is concerned with the process of learning mathematics, rather than the outcome of what mathematics the students knew. Meaning was vitally important because of the emphasis placed on it by the reviewed literature, especially with respect to cognitive development.

In selecting the subjects, students from varying backgrounds who met a number of requirements were sought. In order to be able to compare Ontario mathematics classes with those of another country, the research subjects should have been in Canada long enough to have experienced enough schooling here to make such a comparison meaningful. A consequence of this length of residency is that they should have obtained a conversational level of English proficiency, making the interview process easier. Also, as both positive and negative influences were to be sought and analyzed, it was expected that the research subjects would have had some success in mathematics classes. Finally, as algebra at the grade 9 level was the topic on which the research was based, the research subjects needed to have completed or be then taking grade 9 mathematics.

The English as Second Language teacher at a London public secondary school recruited four students who met all or most of the criteria. Three were girls and one was a boy. All were in grade 9 at the time.

The Four Students

For the sake of confidentiality, all the students have been given pseudonyms. A brief description of each student follows.

Kim was a vibrant talkative young woman who identified herself as being from South Vietnam. She was not sure how long she had been in Canada, but from her responses to the interview questions I determined she had been in Canada just under seven years. Prior to coming to Canada Kim had also lived in Cambodia, Thailand, and one other country whose name she could not remember. She was not sure what her mother tongue is, but spoke Cambodian, English, and some Vietnamese. Nor was she clear who her parents were, as she spoke of being swapped as a child but also spoke of her mother, father and brother in the present tense. Kim often described herself in negative terms, in several cases comparing herself to her brother whom she described as smart and who did not need to take ESL. She regularly spoke about being in conflict with her parents. She said that she first entered a Canadian classroom at the end of grade 2, so grade 3 was her first complete year in a Canadian school. She had completed grade 9 mathematics at the time of her interviews.

Jane was a very soft-spoken young woman from Libya. She had been in Canada only four months, having just arrived from Libya. Jane seemed quite eager to please during our two interviews, bringing in a couple of her Libyan mathematics texts, introducing me to her younger brother (whom she seemed quite proud of as he too had been placed in grade 9), and showing me her latest mathematics test. Her mother tongue is Arabic, and her parents had started teaching her the English alphabet when she was six years old. She began learning English formally in grade 10 in Libya, which was the grade she was in when she moved to Canada. Prior to then her education had been conducted in Arabic. I found Jane the most difficult of the students to communicate with, because of her inability to express herself in a way I always understood and because of her frequent misunderstanding of what I was asking her. She was taking grade 9 mathematics at the time of her interviews.

Dorothy identified herself as being Kurdish, from the Iraqi part of what is known as Kurdistan. She struck me as someone with a strong will and strong opinions, and a willingness to express those opinions, making her a very interesting person to have discussions with. Dorothy had been in Canada four years when I interviewed her. Prior to coming to Canada she had spent four years in a refugee camp in Turkey, and also lived in one other country whose name she could not remember. One of her brothers had died at the camp; she introduced me to another brother and an older sister at her secondary school. Her mother tongue is Kurdish, but she had also learned Arabic and a language she thought was called Suranye. She knew little English before arriving in Canada. Dorothy had completed grade 9 mathematics at the time of her interviews.

Jacob had arrived from Guatemala approximately two years earlier, and had spent all of his life prior to that in Guatemala City - the capital of Guatemala. He appeared to be of Mayan ancestry, as opposed to Spanish ancestry, but that was something he did not confirm. Jacob was very polite and accommodating during our interviews. He did not reveal much about himself beyond the bounds of the interview questions, and his answers to those questions were in general quite brief. His mother tongue is Spanish, and he had learned some English in Guatemala. Jacob had completed grade 9 mathematics at the time of his interviews.

Data Collection

To collect the data from each student, two interviews were conducted. Each lasted no more than forty-five minutes, and was held at the students' school either during their lunch period or after classes had ended. The first interview was guided by a prepared list of questions. The second interview involved the students' attempting a set of mathematics problems. Both interviews were tape recorded and transcribed in their entirety. The students' notes from the second interviews were also kept.

I chose interviews as the method of data collection because, in the words of Bogdan and Biklen (1982), "the interview is used to gather descriptive data in the subjects' own words so that the researcher can develop insights on how subjects interpret some piece of the world" (p. 135). In addition, the reviewed literature that struck me most used the research subjects' own words, and I wanted to attempt similar work in order to have the same powerful and yet personal effect. The actual type of interview I prepared I would term 'moderately structured.' Referring again to Bodgan and Biklen, it allowed me as interviewer the ability to pursue a range of topics while still allowing the student being interviewed opportunities to shape the content of the interview.

Interview One

The first interview was on the student's life up to the time of the interview. It was designed to seek information on culture, language and cognition factors by gathering data which could be used to reflect upon the theories described in Chapters One and Two. There were eleven initial questions. These questions were intended to relate to the variables found in the reviewed literature.

The questions used with each student, and their justification from the literature, follow. The primary literature support was presented earlier in Chapter Two. Note that some questions may initially appear to address one factor or theme but, given the overlapping nature of language, culture and cognition, in fact could address several. It should also be noted that in each case these questions were the starting point for discussion with the students. Additional questions grew out of their responses.

What country did you come from? This question would reveal the student's origins, and lead to further discussion on mother tongue and culture. Myers and Milne (1988) considered home and primary language as factors affecting the learning of mathematics. Vygotsky (1934/1986) and Wilson, Teslow and Taylor (1993) wrote about social motivation as it related to the development of thought, hence my interest in learning about the student's culture. Also pertaining to culture, Leap (1988) and Ascher (1994) spoke of cultural inhibitors which could affect the learning of mathematics.

What memories of school and math class do you have from there? This question continued the discussion on the student's culture, or at least the student's experiences in a learning environment outside of Canada. Again this applies to motivation, and could also reveal the roots of the student's attitude towards mathematics. Taylor (1993) said that attitude was dependent upon culture, socialization and schooling experiences, among other factors.

Who helped you with your homework? Myers and Milne (1988) considered home environment a factor in mathematics achievement. Taylor (1993) wrote of a student's relationships with mentors, role models and family in the development of attitude. This question could lead into further discussion about cultural views of student work.

What language do you speak at home? Home language and primary language were factors considered by Myers and Milne (1988). Clarkson (1992) and Saxe (1988) studied the effects of bilingualism on learning. Vygotsky (1934/1986) felt that thought and word meaning were intimately linked, suggesting that working in a different language could affect one's learning. Skemp (1978) wrote about the "faux ami," where the same word could be used with different meanings. Berry (1985) discussed the structure of a person's language having an influence on his or her cognitive processes. Vygotsky (1934/1986) and Spanos, Rhodes, Dale, and Crandall (1988) wrote about stages in word meaning, with the latter authors saying it could take as long as seven years for a student to become proficient enough in a second language to function effectively in cognitively demanding tasks.

Thus the literature led me to suspect that if, among other things, cultural or ethnic variances in cognitive structures did exist, a student's preference to work in his or her mother tongue would be a clue to those variances.

When you are doing a math problem, do you think in English or in your mother tongue? This question again relates to the connection between thought and language, but was also motivated by Berry's (1985) discussion of culturally distant curriculum when a mathematics student in Botswana thought in English (p. 21). This was the last question to do with what could be called objective data.

What do you think mathematics is? This was the first question to do with the student's perceptions. D'Ambrosio (1985b) wrote of out-of-school applicable mathematics versus school mathematics. He also coined the word ethnomathematics (1990), implying that different cultures had different understandings of mathematics. Nunes (1992) wrote about multiple mathematical systems existing within a single culture.

With this question there was an expectation that, with some probing, a student would reveal that mathematics was done 'differently' in his or her home country.

Do you think your teachers in the other country were strict? This question was intended to open the discussion on differences the student perceived between the functioning of a Canadian classroom and their other classroom experiences. It could uncover clues about attitude and motivation, expose some cultural inhibitors, and perhaps even reveal 'strict' to be a faux ami. Also, as Ogbu (1992) had written of the effects on learning of being in a minority culture, this question could suggest evidence of different treatment of minorities.

What do you find different in math classes in Canada? This question could potentially reveal a huge quantity of data pertaining to the learning of mathematics. Presmeg (1988) wrote about living with the influence of several cultures. This question could also uncover data on cultural inhibitors, minority cultures, and other mathematics, as well as on the connections between thought and language and the question of language proficiency already mentioned.

What do you find difficult about mathematics as it is taught in Canada? This question continues from the previous one, but could also produce data for two important concepts. Given that the students had come from other countries, I anticipated the issue of language would be raised. In addition to the theories of stages of meaning and language proficiency mentioned earlier, Sierpinska (1993) looked at the development of concepts as it related to the learning of mathematics, and said one needs a knowledge of one's own thinking processes. This might be more difficult in a new language.

What, if anything, do you like about mathematics? This question is another pertaining to attitude.

Do you do anything you perceive of as mathematics outside of the school environment? This question could provide data for D'Ambrosio's (1985) and Nunes' (1992) theories of different types of mathematics, and for Presmeg's (1988) discussion of living under the influence of multiple systems. It was used in conjunction with the question, "What do you think mathematics is?"

Interview Two

The second interview was mathematics specific. Each student was given a selection of questions from the Chelsea Diagnostic Mathematics Test for algebra. Instead of simply answering the questions in writing, I asked each student to talk through each question as he or she was doing it so that I could try to understand his or her thinking. Also, inspired by Vygotsky's (1934/1986) concept of the zone of proximal development, I tried - if a student was stuck with a problem - to gently prompt him or her to see if and how an answer could be obtained.

The specific questions selected from that test for use in the second interview are found in Table 4. The questions were chosen based on Küchemann's (1981) analysis of Chelsea algebra test results. In fact, they are the exact questions he chose to illustrate his six categories of children's interpreting of letters: letter evaluated,

Table 4

Selected Chelsea Diagnostic Test Questions, with Level and Category

Question			Category	Level
3.	Which is the larger, 2n or n+2?		Letter as Variable	4
4.	4 added to n can be written as n+4. Add 4 onto each of these:		Letter as Specific Unknown	
	(i)	8		N/A
	(ii)	n+5		N/A
	(iii)	3n		3
	n multiplied by 4 can be written as 4n. Multiply each of these by 4:			
	(iv)	8		N/A
	(v)	n+5		4
	(vi)	3n		N/A
5.	(i)	If a+b = 43, a+b+2 = ...	Letter Not Used	1
	(ii)	If n-246 = 762, n-247 = ...		N/A
	(iii)	If e+f = 8, e+f+g = ...		3
6.	(i)	What can you say about a if a+5 = 8	Letter Evaluated	1
	(ii)	What can you say about b if b+2 is equal to 2b		N/A
11.	(i)	What can you say about u if u = v+3 and v = 1	Letter Evaluated	2
	(ii)	What can you say about m if m = 3n+1 and n = 4		2

13.	a+3a can be written more simply as 4a. Write these more simply, where possible:	Letter as Object	1
	(i) 2a+5a =		3
	(ii) 2a+5b =		N/A
	(iii) (a+b)+a =		2
	(iv) 2a+5b+a =		4
	(v) (a-b)+b =		N/A
	(vi) 3a-(b+a) =		N/A
	(vii) a+4+a-4 =		3
	(viii) 3a-b+a =		N/A
	(ix) (a+b)+(a-b) =		
14.	What can you say about r if r = s+t and r+s+t = 30	Letter Evaluated	3
16.	What can you say about c if c+d = 10 and c is less than d	Letter as Generalised Number	3
18.	When are the following true - always, never, or sometimes?	Letter as Generalised Number	
	(i) A+B+C = C+A+B		N/A
	(ii) L+M+N = L+P+N		4
22.	Blue pencils cost 5 cents each and red pencils cost 6 cents each. I buy some blue and some red pencils and altogether it costs me 90 cents. If b is the number of blue pencils bought, and if r is the number of red pencils bought, what can you say about b and r?	Letter as Object	4

letter not used, letter used as an object, letter used as a specific unknown, letter used as a generalised number, letter used asvariable. My hope was that the insights Küchemann described with regards to these particular questions could help shed light on the cognitive workings of the students participating in my research.

Method for Analysis

The data from the first interviews was read and re-read to look for emergent themes. Bogdan and Biklen wrote that this kind of approach would allow one to "attempt to gain entry into the conceptual world of [the] subjects ... in order to understand how and what meaning they construct" (p. 31). While the theoretical constructs drawn from the reviewed literature are helpful, "[i]nterpretation ... is essential" (Bogdan and Biklen, p. 34).

To allow for this interpretation, the data from the first interviews were divided into 'meaning units.' Each interview question was motivated by constructs drawn from the reviewed literature, and each meaning unit corresponded to one construct. A systematic method was needed in order to extract the individual units of meaning from the transcripts. First the transcripts were read to relate the responses to each of the eleven interview questions. This resulted in nine blocks of text responding to the first question, nine for the second question, six for the third question, five for the fourth, four for the fifth, four for the sixth, ten for the seventh, fifteen for the eighth, seven for the ninth, four for the tenth, and six for the eleventh question. The fact that there are in some cases more blocks of text per question than there were students is a consequence of the nature of the interviews; a block is an uninterrupted subset of a transcript answering one of the eleven initial questions, but due to the ebb and flow of the discussion sometimes a complete answer took several attempts to obtain.

The blocks, or responses to the questions, were read again and again until I could identify meaning units within the blocks. The meaning units I identified were not restricted to the constructs which informed each of the eleven questions; additional unanticipated constructs also arose. An example of a meaning unit comes from part of Jacob's response to the question 'What do you find different in math classes in Canada?'. The following was identified as a block of text pertaining to that question:

Mark: When you came to Canada, when you first went to a Canadian school, did you go ... was there anything that you thought, "This is very strange"? You know, that they are doing things ... either teaching differently or dressing differently or have different rules ...
Jacob: Dresses, yeah. They are different. Because we don't dress like this. We use uniforms.
Mark: Oh, you had to wear uniforms?
Jacob: Yeah, we had to buy uniforms.
Mark: In the College and the Private School, or both?
Jacob: Yeah, the two of them.
Mark: And do you think that's good to wear a uniform? Or do like dressing in your ...
Jacob: Well, sometimes I like to use uniform.
Mark: Why is that?

Jacob: Cause you show your clothes. You don't have nothing to wear when (?) ... Or, when somebody sees you, they see that you are from that school. Cause you wear uniform.

I considered the portion of this block where Jacob speaks about wearing a uniform,

Jacob: Well, sometimes I like to use uniform.
Mark: Why is that?
Jacob: Cause you show your clothes. You don't have nothing to wear when (?) ... Or, when somebody sees you, they see that you are from that school. Cause you wear uniform.

to be a meaning unit referring to motivation - specifically his pride in his school. Some meaning units were longer than this example, and others were a single line.

Of the constructs identified which were not anticipated in the design of the question, most of these were constructs already described in the literature review. However, after some reflection I added one new construct which I have termed 'Stability.' Table 5 illustrates the breakdown of meaning units by student.

Regarding interview two, the students' solutions to the Chelsea algebra questions which had a level associated with them were scored using the scheme provided with the tests. Each solution was assigned a code, as follows:

- Code 0 - not answered
- Code 1 - correct
- Code 2 - ambiguous
- Code 3/4 - letter evaluated
- Code 5/6 - letter as object

Table 5

Meaning Units Per Construct by Student, First Interview

Construct	Dorothy	Jacob	Jane	Kim	Total
	Student				
Language					
• proficiency	6	3	1	3	13
• bilingualism	5	5	4	4	18
• words or symbols used	3	1	0	1	5
Culture					
• culturally different curriculum	0	0	0	0	0
• multiple systems	9	5	3	2	19
• cultural inhibitors	6	10	5	5	26
• majority versus minority culture	5	0	0	0	5
Cognition					
• word meaning	2	1	3	2	8
• level of understanding	0	0	5	1	6
• motivation and attitude	22	14	14	33	83
• concepts	0	0	0	2	2
Home Environment	4	1	3	9	17
Stability	2	0	0	7	9
Total	64	40	38	69	211

- Code 7 - letter not used
- Code 8 - premature closure
- Code 9 - none of the above.

Codes 3 to 9 refer to *how* the student reached an incorrect answer, allowing some insight into his or her thought processes. Codes were assigned for the student's original solution, and the solution obtained after being prompted. These codes were then assembled in a table to see if any insight into the students' thought patterns could be gained from analyzing the results and comparing with Küchemann's work. That table is reproduced in Table 6.

In addition, the tape recordings of the students' answers to my questions and promptings during the second interviews, as well as their written work from solving the Chelsea questions, were searched for additional meaning units to add to those of the first interviews. I anticipated that the transcripts and written work from these interviews could relate to Vygotsky's (1934/1986) stages of meaning, Sierpinska's (1993) development of concepts, Skemp's (1978) ways of understanding, and Spanos, Rhodes, Dale and Crandall's (1988) mathematics register. The meaning units I identified, itemized in Table 7, were added to the list of meaning units prepared from the first interviews.

The complete list of meaning units was then reread and the meaning units classified as either interesting or

Table 6

Chelsea Solution Codes Per Question by Student

	Student			
Question	Dorothy	Jacob	Jane	Kim
3.	8	8	8	9, prompted 8
4.(iii)	7	1	1	7, prompted 9
4.(v)	7	7, prompted 2	2, prompted 1	9
5.(i)	1	1	1	9, prompted 1
5.(iii)	9	1	1	9
6.(i)	1	9, prompted 1	1	1
11.(i)	0, prompted 1	9, prompted 1	1	1
11.(ii)	1	1	1	1
13.(i)	9, prompted 1	1	1	1
13.(ii)	1	9, prompted 8	1	9
13.(iv)	1	1	--	9
13.(v)	9	9, prompted 1	2, prompted 1	--
13.(viii)	--	--	--	9
14.	1	0, prompted 1	0, prompted 1	9
16.	0, prompted 8	0, prompted 1	1	--
18.(ii)	8	8	0, prompted 2	8
22.	0, prompted 9	9	9, prompted 5	0, prompted 9

Table 7

Meaning Units Per Construct by Student, Second Interview

Construct	Student: Dorothy	Jacob	Jane	Kim	Total
Language					
● proficiency	4	5	0	0	9
Cognition					
● word meaning	0	0	1	1	2
● level of understanding	4	3	2	12	21
● motivation and attitude	0	0	0	1	1
● concepts	3	3	2	8	16
Total	11	11	5	22	49

uninteresting with respect to my investigation. This process continued until I felt I had a good understanding of how the students' responses fitted in with the theories I was exploring. The results of this analysis are given in Chapter Four, and my thoughts on other unanticipated themes are in Chapter Five.

CHAPTER FOUR

Results

In the preceding chapter the methodology for obtaining and analyzing the data was described. In this chapter the results of that activity are presented using the three headings of language, culture, and cognition. The use of these headings follows the way the reviewed literature was presented, and is not meant to imply that the headings will be looked at in isolation. To facilitate reporting, each heading is further subdivided. The meaning units derived from the data of the first and second interviews provide the basis for the results. Excerpts drawn from those units will be used to illustrate certain points. In those excerpts the name 'Mark' identifies the researcher; the other names are the pseudonyms given to the students in Chapter Three.

Language

From the analysis of the literature reviewed in Chapter Two, three themes in which language could affect the learning of mathematics emerged. The interviews produced data for all of those themes: proficiency in the working language, words or symbols used by a language, and bilingualism.

Proficiency in Working Language

A student in one of Ontario's English-speaking mathematics classrooms needs a working proficiency in the language. In addition, Spanos, Rhodes, Dale, and Crandall (1988) suggested that a certain amount of proficiency is required in the language of mathematics - the mathematics register. Thus the theme of proficiency was explored with each of the four students.

All of the students required some degree of assistance with reading and understanding the Chelsea Test questions. This may simply have been due to a lack of familiarity with the English language, but could also be seen as suggesting they had not completely developed their mathematics registers. In other words, they may have recognized the words but not understood their context within the mathematics problem.

Regarding proficiency in English, all four students reported that they thought in English and spoke English at home. However, there were variations amongst them as to the quality and quantity of English used.

Dorothy said that she had to think in English, otherwise she would blurt out Kurdish words. She had had little formal English language training before coming to Canada, and seemed to have trouble functioning in English when she had several concerns on her mind. The latter point came to light as she digressed from her answer to my question of how Canadian teachers compared to her other teachers.

Excerpt 1:

Dorothy: He doesn't go slow on you. And he goes, "Okay, tomorrow's the test." "Like, what's it on?" "Tomorrow's the test on the section that we studied." And then we have to go through all the section to just try to memorize it. And then sometimes we have homeworks from other classes too. And we have to try and remember. Once I had a speech to do. And I had to remember Business because we had a test there. So I went up to the ... up in the front of the class. And I was saying my speech. But I couldn't remember anything because Business was in my head. I studied that a lot. It was in my head. I couldn't ... And then I had to go sit down. I couldn't say my speech. Because of Business ... The Business was keep coming in front of my eyes and in my mouth.

Mark: When ... when you are thinking about your speeches or your Business, do you think in Kurdish and then translate it into English or are you thinking in English?

Dorothy: I think in English. I can't think in Kurdish.

Mark: No?

Dorothy: No. I do that I'd probably say it to the class.

Dorothy clearly was still making conscious efforts to communicate in the language.

Kim, on the other hand, spoke English seemingly without effort, and in my estimation was the most proficient English speaker of the four students. She said that her father had taught her some English, and given the length of her stay in Canada she had had the most immersion in the language. Kim reported that she thought in English, but spoke a mixture of English and Cambodian at home.

Jacob reported that he thought both in Spanish and in English. During the two interviews, however, he did not indicate that he had switched to thinking in Spanish, so there was no opportunity to explore the circumstances in which he would need to think in Spanish. At home he generally spoke Spanish, although he occasionally used English with his brothers. He did not understand several of the Chelsea Test questions until prompted or assisted with interpretation.

Despite having had little formal English training, Jane also reported thinking in English. She had been introduced to the language by her parents while in Libya. This fact took me several attempts to determine, as I often misunderstood what Jane was trying to tell me. The following excerpt gives some background to her English 'lessons', and also serves to illustrate the difficulty we had in communicating with each other.

Excerpt 2:

Mark: You've only been speaking English for a year? For one year you've been ... you said you started learning English in grade 10?
Jane: No. At home my parents teach me how to speak and read English.
Mark: Oh. I thought you had just begun learning English in grade 10, and that was, like, one year.
Jane: Yeah, because we were in America.
Mark: Oh, I see ... Okay, now I'm getting a little confused. You were in grade 10 in Libya ...
Jane: Yeah.
Mark: ... and then you went to Canada ...
Jane: Canada.
Mark: ... just in January. Of this year? January 1996?
Jane: Yeah.
Mark: But your parents were speaking English at home in Libya? Is that ...
Jane: Yeah. They know English. They know lots of English.
Mark: So when you were a small baby they started to teach you? Is that right?
Jane: When I was six years' old I (unclear) the alphabet. A, B, C, D. Somehow (unclear) hard

work, or just they teach me.
Mark: Do many people in Libya speak English, or is that ... unusual?
Jane: Not so.

Given this excerpt and others from Jane that will follow, it is not surprising that she acknowledged the only thing she had difficulty with in mathematics class in Canada was the words.

Thus proficiency did appear to be an important, if not surprising, factor in the students' mathematics learning experiences.

Bilingualism

Also pertaining to language was the theme of bilingualism. The reviewed literature had suggested that proficient bilingual students could have advantages with respect to learning mathematics. As it turned out, the analysis of the interviews conducted with the four students did not support this effect.

As noted earlier, all the students in my study were bi- or multilingual to some extent. However, it appeared that the least competent English speaker - Jane - was the most successful of the four students at mathematics, and the most competent English speaker - Kim - was the least successful. I base this conclusion on my experiences with each student during the second interviews. With respect to the theories of the effects of bilingualism on mathematics achievement, this observation initially appears to contradict Saxe (1988), who had concluded that growing up bilingual positively influences some aspects of a child's cognition about the arbitrariness of numerical conventions, and Clarkson (1992), who had theorized that students competent in both English and their mother tongue had an advantage in mathematics achievement over students competent in just one language.

Perhaps other factors came into play which countered the effects of bilingualism. More will be said on this in the summary section.

Words or Symbols Used in Particular Languages

The third theme to explore with respect to language was words or symbols used by a language. The reviewed literature suggested that the number words or ways of symbolically representing numbers in some languages can either facilitate or hinder arithmetic competence. With four students from such diverse backgrounds it was conceivable that each could have been exposed to different number systems that affected their learning of mathematics once they came to Canada. Two of the four students had in fact been exposed to different ways of writing or expressing numbers.

Jacob was exposed to Mayan numbers while at school in Guatemala City. He had to learn their numbers for one to twenty. However, he said that this was presented as history, and that the Mayan numbers were not actually used in any computations. Nor were they used in daily life in the city. The numerals and symbols he used in school were identical to those used in Canada.

Dorothy did learn a different way of expressing numbers, and it did affect her learning of mathematics. Kurdish numerals have, in some cases, the same appearance but different meaning to the Arabic numerals used in Canada. I asked Dorothy to write out the Kurdish equivalents of some Arabic digits, which are given in Table 8.

Table 8

<u>Kurdish Equivalents of Some Arabic Numerals</u>

Arabic	Kurdish
0	.
1	1
2	<
4	3
5	0

The Arabic number for one hundred and twenty five, "125", would be written in Kurdish as "1<0". Conversely, if Dorothy had written "1<0" in Canada, her teacher would have read it as "one is less than zero," an incorrect statement. The number ten would be written as "1.", which would lead to problems when a sentence ended in the number 1 followed by a period. Finally, a decimal number like three decimal zero one, written in Arabic as "3.01", would mean four thousand and fifty one in Kurdish.

The different values attached to the numeric symbols did affect Dorothy's learning experiences. This next excerpt illustrates her difficulties when she first arrived in a Canadian mathematics classroom.

Excerpt 3:
Mark: So did you find it difficult, then, changing from this kind to Canadian writing?
Dorothy: Yeah. Zero in our language it's a five. It's a five, zero. It's a five.
Mark: That means five?
Dorothy: Yeah. And I didn't understand it. So when I tried to calculate I did it all wrong. And right here? Where it was a three, I thought it was a four.
Mark: That would be difficult! ... That's kinda neat ... So did they teach you how to multiply and that, in your own language?
Dorothy: No. We were doing adding and subtracting.
Mark: Oh, okay. So you learned to multiply and stuff like that when you got to Canada?
Dorothy: Uh huh. It was so hard. But I managed it.
Mark: So you had to learn the different numbers too?
Dorothy: And alphaber ... alphabet. I didn't even know anything about alphabets. When she was saying it I didn't even know what they were saying.

Dorothy had to adjust her symbol definitions, or at least keep multiple definitions, in order to adapt.

Summary

My exploration of the students' mathematics learning experiences implied that language does have an effect on the learning of mathematics. My discussions with the students gave support for a link existing between proficiency in the working language and mathematics achievement. Each student's ability to answer the Chelsea Test questions was limited by his or her knowledge of the language, and perhaps by the size of his or her mathematics register. There also appeared to be an indirect link to culture or ethnic origin, as each student had received a different level of exposure to English before arriving in Canada, and thus had an advantage or disadvantage in terms of proficiency.

Levels of exposure to mathematics could perhaps help explain why bilingualism did not appear to be an important factor. Jane's mathematics learning experiences before coming to Canada seem to have been the most rigorous of the four students, giving her an advantage in experience and familiarity with the material which may have countered the fact that she seemed the least bilingual of the students. Other factors could also have affected the investigation of bilingualism. For example, Saxe (1988) did say that a limitation of his study was the criteria used by researchers to define bilingualism. Thus it is possible the four students did not fit into the particular definition of bilingualism required for a particular theory. In addition, other factors may have had stronger effects than those of bilingualism. Perhaps the fact that three of the four students had used a common numerical system, despite their different languages, countered the literature's predicted effects about dealing with the arbitrariness of numerical conventions. Overall my research suggests that this theme needs to be studied further.

Cultural or ethnic factors did seem to have a negative effect on Dorothy's mathematics learning experiences, with respect to the language words or symbols theme.

What I am most struck by as a result of this exploration into language are the little glimpses I got of the connections to cognition. For example, in Excerpt 1, Dorothy, somewhat similarly to Berry's (1985) Botswanan university student, is indicating something about her cognitive workings as she may actually think in two 'languages' but restrict herself to thinking in English for school work. There is much more to this factor than merely speaking the words.

Culture

The four students participating in this study came from four different cultural groups. It was conceivable that growing up in North Africa (Libya), Southeast Asia (Vietnam / Cambodia / Thailand), Latin America (Guatemala) or the Middle East-Eastern Mediterranean region (Iraq / Turkey / Kurdistan) could have affected the way the students performed in a Canadian mathematics class. The reviewed literature suggested four themes with which to explore culture as it pertained to the learning of mathematics in an Ontario classroom: culturally different curriculum, multiple systems within a single culture, cultural inhibitors, and majority versus minority culture.

The results of those explorations follow.

Culturally Different Curriculum

For curriculum to be culturally different, it has to be implemented in a manner foreign to one's own cultural ways of expression. For example, Berry (1985) suggested that students from oral-based cultures would have difficulties with written materials because of the vastly different ways each method of communicating would use to express ideas. In my research this theme was probed by asking the students to explain what they found different or strange about their Ontario classroom experiences compared to their earlier experiences in their 'home' country.

Nothing resulted from that questioning that indicated the four students were affected by a culturally different curriculum. All of them were from cultures which included the tradition of writing, which made it impossible to explore Berry's theory. Furthermore, three of the four students - Kim, Jane, and Jacob - reported that when they first entered a mathematics class in Canada, they had in fact already seen all or most of the material before. Consequently this present research can only suggest that the theme of a culturally different curriculum needs to be explored further.

Multiple Systems Within a Single Culture

The second theme dealing with culture had to do with multiple systems of mathematics within a single culture, and how they may have affected the students' mathematics learning experiences. For example, if one has learned to express distance in units of time when talking to people on the street - you are twenty minutes from the bus station - and then has to calculate distance in terms of metres and kilometres in school, there could be some effects on how one performs in mathematics class.

In an attempt to uncover multiple views of mathematics, the students were asked to define what mathematics was and were also asked about any practises they had seen in their own countries which could be called mathematical. Dorothy and Jane said that mathematics was strictly a school activity, although Dorothy also acknowledged performing calculations elsewhere.

Excerpt 4:

Mark: Do you ever use ... stuff at home or when you're outside of school that you think is mathematics, or is mathematics just a school thing?

Dorothy: No, I ... um ... I don't know.

Mark: Like, you say mathematics is numbers and adding and multiplying. Do you ever do that kind of stuff at home? Or when you are grocery shopping or looking for clothes?

Dorothy: Oh, clothes, yeah. I don't ... I don't take that as mathematics that much.

Mark: Oh okay, you just do it ...

Dorothy: And when I'm in a class its ... that's what's left in the class. I don't take ... I don't carry with me. Not usually.

Kim and Jacob could also identify some of their 'school' mathematics being used outside of school, without adding a disclaimer like Dorothy did in Excerpt 4. None of the students had an unusual definition of mathematics; each defined it more or less in terms of the activities he or she performed in class, such as 'brain work,' 'big tests,' 'numbers,' 'multiplying and subtracting.'

Dorothy was the only student to report seeing what could be a cultural mathematical practise. This arose during the discussion on using mathematics outside of the classroom:

Excerpt 5:

Mark: Well let's say you wanted to buy, I don't know, did you get fish in the camp?

Dorothy: Yeah.

Mark: Let's say they said the fish, one fish is 25 cents. And you say, well, I want seven fish. So how did they calculate how much you should pay them so that they know you gave them the right money?

Dorothy: Good question!

Mark: Do you know that?

Dorothy: I think ... Well, when I was shopping with my Mum I never saw ... I didn't never saw any calculation or any thing like that. They did them out of their head. Or on a paper.

Mark: Oh, they had paper? I was wondering if they had any tricks to do these kinds of things, but ... They would actually sit down and say ...

Dorothy: There was a trick. They used their fingers, but ... My brother would try to teach me but I wouldn't know ... what he does. There are some things that they use, but I don't ... I wasn't that old to remember them.

Because Dorothy did not remember the actual calculating act, I could not pursue this theme further.

Instead of being exposed to a different mathematical system, Jane was taught incorrect or improper rules for the same system as it is used in Canada. This came to light when we were looking through her Libyan text books. I asked her to demonstrate how she calculated a square root, which ended up with us discussing how to multiply two decimals together.

Excerpt 6:

Mark: And all this other stuff you've done ... You did square roots? Did you have to calculate these?
Jane: Not the calculator, no. (unclear).
Mark: So if I ask you the square root of 2, did you have to memorize this?
Jane: No.
Mark: So you couldn't just work it out with a pencil? You had to memorize?
Jane: Yeah ... I don't understand.
Mark: Okay ... Let's say I gave you this one [from Libyan text]. Now do you ... what would you do to find that?
Jane: [describes algorithm]
Mark: Oh, so you just try to guess which one works. Is that right?
Jane: [keeps working on problem] ... It's not like this.
Mark: Yeah, I understand what you're saying. Actually, how did they teach you ... so 2.5 times 2.5, now ... how did they teach you to multiply that?
Jane: [demonstrates algorithm, which puts the decimal in the wrong location]
Mark: Now why did you put it there? How ...
Jane: Because when I was in grade 4, we had this book, we had (unclear), the teacher and when you get multiply (unclear).
Mark: Hmm. That's interesting. So you had to memorize this rule?
Jane: Uh huh.
Mark: Did you ever see this rule? [demonstrates method that gives correct solution] ... So which is correct do you think? This one [hers]? Or this one [mine]?
Jane: No. This one [points to mine] ... They teach us this one [points to hers].
Mark: I guess ... They teach you this one. But now you tell me this one is correct ... But this is the one they teach you ... So I don't understand. Why ... If you know that this one is correct ...
Jane: Just this year.
Mark: Oh, okay. So ... you only learned that this year?
Jane: Uh huh.
Mark: So other times you would have written this one down?
Jane: Yeah. (unclear) this one. My Dad said to my ... said to Mrs. **** [the ESL teacher] the teacher

in math not good.

This incorrect method could be a consequence of rote learning, but regardless of the reason for being taught the wrong algorithm it is clear that it briefly affected Jane's work in the Canadian mathematics classroom. It also raises the question of how many other algorithms she had learned incorrectly.

Thus multiple systems of mathematics appear to have been experienced by some of the students. What is questionable is whether those experiences affected them in any significant way.

Cultural Inhibitors

Cultural inhibitors were considered as the third theme addressing culture. Inhibitors would include anything within a culture which makes it harder for a student to get the full benefit of the learning environment created in an Ontario classroom. This theme was explored both explicitly and implicitly.

None of the students gave examples of things they felt inhibited their participation in mathematics class in Canada. However, by inference two of them implied that there were inhibitors.

Jacob described what could be interpreted as a cultural inhibitor when he was telling me what would strike a Canadian visitor to Guatemala City as different.

Excerpt 7:
Mark: Do you know what I mean by culture?

Jacob: Culture, yeah.
Mark: Are the cultures the same, or are they different? ... Like, if I was to go to Guatemala City, where you used to live, would things work the same way they work here in London? You know, could I go talking to people, if I could speak Spanish ...
Jacob: No.
Mark: No? What's different?
Jacob: Like, here you find somebody who's walking, you say, "Hi, hello." Over there, they don't talk.
Mark: They don't talk? You mean they just pass each other in the streets?
Jacob: Yeah, their friends, yeah.
Mark: Okay. But they don't talk in the streets, you mean?
Jacob: Yeah, that's right. Only if they are working on the streets. Like they sell something, they put ... prices on the streets.
Mark: Why don't they talk? Do you know? No idea?
Jacob: No idea.
Mark: So is there anything else you think I'd find different if I went to Guatemala City?
Jacob: ... Let's see ...
Mark: Like, in some places, if you were to go to Arabia, you know, the Middle East ... the women always have to be covered up. Are there any clothes that people don't allow ...
Jacob: No. The same, the same.
Mark: The same? And the buildings look the same? The cars look the same?
Jacob: No, the cars look a bit more ... some of the rich people, yeah, got better cars.

The only thing Jacob noted as being different was the way people spoke to each other. This suggests Jacob's culture could have constraints on when you speak and to whom you speak. As noted in Chapter Three, I had a sense of this from his way of answering my questions. It is possible a constraint such as this could have affected Jacob's participation in a classroom setting where he was expected to ask questions of the teacher and respond to the teacher's questions. Jacob did not comment on that as an effect.

Dorothy found both the manner of dress and behaviour of students very different when she arrived in Canada. Her feeling about these points may have initially served to inhibit her interactions with her Canadian peers, as can be inferred from the following two excerpts.

Excerpt 8:

Mark: Do you know what the word culture means?

Dorothy: Yeah.

Mark: Like you said when you came to Canada the people were different. How do you see the people were different when you came to Canada?

Dorothy: In my culture, they wear ... You know they seen they wear turbans on their head? But I never wore them, so I wouldn't know what it felt like under it. Some people they wear that. And then I come here, like, most people are almost naked! And I didn't know ... I felt embarrassed when I saw that. I go, how could they stand wearing those clothes?

Mark: So you are used to wearing clothes that cover all of your body, right? You didn't wear shorts, or short skirts, or go around with bare arms?

Dorothy: We go around with bare arms. Well, our family did. I don't know others. We wear shorts, but not really short stuff like this. And we have nothing on our heads. And one things I didn't get used to, wearing makeup. Cause in our country we wear it only for celebrations. And I came here, like, people wear it even when they go to sleep sometimes. And I found that very strange.

Excerpt 9:

Dorothy: [I]n Turkey I didn't care about what they did, because they were, they speaking my language and all that. And people here ... people there they didn't stare at me if the teacher yelled. They kept doing their work. Cause the teacher said, "If I yell at someone I want you not to pick up your head. Just do what you're doing." But here everybody stares at you and laughs at you.

Mark: Now, do you think that's the students' problem? Do you think the students are ruder than they were when you were with your own people in Turkey?

Dorothy: Students here are rude ... I think it depends on what they see around them. The society. They

> probably act weird so they think its nice and good and its cool, so they do it. Same.

It is clear from Excerpt 9 that Dorothy did not agree with her peers' behaviour. Thus Dorothy's reactions to Canadian culture possibly inhibited her initial interactions with her peers at a social level. This inhibition could also have affected her mathematics learning experiences, as she would have been required to interact with her peers in class too. However, evidence of this happening did not come out of our conversation; she was more interested in telling me what was wrong with her mathematics teachers - which had nothing to do with dress or manners.

Hence both Jacob and Dorothy indicated their cultures contain factors which had at least the potential to inhibit their mathematics learning experiences.

Majority versus Minority Culture

The final theme dealing with culture has to do with the student being a member of a majority or a minority culture. Ogbu (1992) wrote of voluntary and involuntary minorities, and said that one's motivation and attitude towards learning depended upon which of those two minority types one was a member of. Thus the present research included a consideration of whether any of the students had belonged to what could be classified as a minority group, and how that group membership had affected their learning experiences.

All of the students were voluntary immigrants to

Canada. However, Dorothy was in a position to provide insight into the question of the effects of membership in an involuntary minority because of her refugee experiences. This became apparent during her recounting of events at the refugee camp in Turkey:

Excerpt 10:
Dorothy: I stayed there ... First the conditions were good there. But then after it kinda got bad. And they poisoned us with bread once. Some people died. (unclear). And then we would get food every month. And fruits and all that we would get every month in the summers, but otherwise we didn't have them.

Excerpt 11:
Mark: And did you have it (mathematics classes) for all four years, did you go to school or just for some of them? Because you said that things got bad. When things got bad, did classes stop?
Dorothy: They stopped them for a bit. And then we started them again. But ... I don't know ... there was once we didn't have schools for two months. Or so. I don't know. We didn't have them. But then, when we went back there were a lot of other work. And I think we caught back a subject, but I can't remember. What was it?

Dorothy did not say who "they" were, but it became clear through the two interviews that the Kurdish people experienced friction with both the Iraqis and the Turks. Also, with reference to Excerpt 11, it should be noted that in the refugee camp school normally ran all year, so the two month break was unusual.

The refugee camp experience may have helped Dorothy strengthen her national identity, but other than missing two months of classes there is no indication that her minority status there affected her learning of mathematics.

Thus my research did not find any support for Ogbu's theory of the difficulties associated with being a member of an involuntary minority. The students and I did not explore the benefits of their being in a voluntary minority in Canada.

Summary

My explorations with the students of the culture themes did not indicate they had experienced any culturally specific positive effects on their learning of mathematics. One theme, culturally different curriculum, could not be explored with the four students. It remains in need of further study, as does the theme of minority group membership. In Jacob and Dorothy's cases, their cultures might have imposed some difficulties on their mathematics learning experiences, specifically in terms of inhibiting their participation in class. However, similar or identical inhibitions might be experienced by students born and raised in Ontario.

The exploration relating to multiple systems suggested that, based on Dorothy's shopping with her mother, people do practise what D'Ambrosio might call ethnomathematics. There was also the issue of rote learning as another system of mathematics, but this cannot be considered a cultural or ethnic factor as it too is 'practised' here in Ontario. The four students did not appear to be affected by any culturally or ethnically based systems of mathematics. In fact, they seemed no more affected by multiple systems of mathematics than an Ontario-born student would be.

In addition to analyzing the constructs from the literature, my reflecting on the students' behaviours during the interviews has made me wonder if Dorothy, Jacob and Jane's cultures influenced the way they interacted with their Canadian mathematics teachers, and consequently affected their mathematics learning experiences. Dorothy might appear to some people as somewhat forceful or perhaps slightly aggressive; certainly during our two interviews she was very candid and expressed strong opinions. A sense of her manner of communicating comes from her response to my asking her if teachers seemed different in Canada.

Excerpt 12:

Dorothy: I get nervous more when I do my tests here. Because most teacher here, they ... I got my [subject x] teacher. I'm not gonna say the name, so ...

Mark: It doesn't matter. No one's going to ... Your [subject x] teacher won't hear this, so it doesn't matter.

Dorothy: He ... I'm an ESL student, and I have another friend. She's ESL too. And we're with the regular people are learning about [subject x]. And he doesn't write it on the board, and he's gonna ask us questions. And he's gonna say it out of his head, and we have to write it down. I tell him to go slow but he doesn't go slow. So that makes me very mad. I swear to God! If I had his permission I would kill him.

Mark: Yeah, well ...

Dorothy: He doesn't go slow on you. And he goes, "Okay, tomorrow's the test." "Like, what's it on?" "Tomorrow's the test on the section that we studied." And then we have to go through all the section to just try to memorize it. And then sometimes we have homeworks from other classes too. And we have to try and remember.

It seems probable that some teachers would find this manner of expression inappropriate or unacceptable.

Jane on the other hand would probably be a mathematics teacher's ideal student. Her enthusiasm and willingness to please during the interviews has already been described in Chapter Three. Jane also seemed to have very clear ideas of how teaching should properly be practised, as is illustrated when I asked her about teachers she had liked:

Excerpt 13:

Mark:	Um ... Were there any teachers you had in Libya that you thought were very good teachers? That you liked? That you thought were good teachers?
Jane:	Just one teacher.
Mark:	What made that teacher a good teacher.
Jane:	(?).
Mark:	Um ... You had one teacher from Libya that you thought was a good teacher. So there must have been something about that teacher that made you think this one is a good one.
Jane:	Yeah, because my teacher in Libya, she was like my teacher here. They write on the blackboard, they (?) how you can write, they give you class work, homework, homework, and on the blackboard how you can check it. (?).
Mark:	I see. So you think she was good because she did all these things to make sure you knew ...
Jane:	Yeah.

Jane seemed to be affirming a certain teaching style, which she had experienced in Ontario.

Jacob would appear to fall somewhere between the interacting styles of Dorothy and Jane. As I have discussed earlier, Jacob seemed to have constraints on when and to whom to speak. These constraints would not likely antagonize his mathematics teachers, but they might prevent him from fully interacting with them - to the detriment of his mathematics learning. However, further exploration is needed in order to investigate whether these mannerisms of Jacob's, as well as those of Dorothy and Jane, are in fact cultural or unique to the individual.

Cognition

Vygotsky (1934/1986) recognized that our thoughts and the ways we know and express or vocalize those thoughts are tied together in a complex way. The review of the literature suggested exploring four themes to study the connection between thought and language: word meaning, level of understanding, motivation and attitude, and concepts. The interviews with the four students allowed each of these themes to be probed to some degree, although without the anticipated links to culture and language.

Word meaning

The theme of word meaning was meant to allow an exploration of how word meaning mediated thought and language, and search for any cultural or ethnic effects on this mediation. Suggestions of this mediation were sought both explicitly with specific interview questions and implicitly with the analysis of the responses to all the questions.

I could not link word meaning explicitly to any of the students' cognitive processes, despite the fact that all of the students were non-native speakers of English and had received formal schooling in another language. All of the students reported that they thought in English when doing mathematics problems, with the exception of Jacob. Jacob said he thought in both Spanish and English, and did not express a preference for either language. No explicit opportunity arose to pursue with a student the effect of different word meanings. Dorothy did speak of the Kurdish numerals, and her confusion when she first encountered Hindu-Arabic numerals in Canada. However, despite the fact that identical symbols were used in each numeral system but to represent different numerical values, Dorothy did not indicate this fact was a serious impediment to her mathematics learning experiences.

When the issue of word meaning did arise during our conversations, it was with respect to the meanings or particular understandings of English words such as 'subtraction' or 'sometimes.' These occurrences appeared to me to be nothing more significant than a student expanding his or her grasp of the English language; none of the occurrences seemed to have anything to do with any sort of conflict with a pre-existing cognitive structure.

With respect to word meaning in terms of relational versus instrumental understanding, there was a suggestion of the latter from both Kim and Jane. Both had experienced being physically punished for giving incorrect answers while in elementary school in other countries. Jane also told about how teachers had simply written out correct answers for the students, as opposed to explaining them. In both cases these facts suggest rote learning and instrumental understanding. However, it should be noted here that, with the exception of the 'corrective' measures used, the experiences of Kim and Jane would probably sound familiar to some students born and educated in Ontario.

Thus there is little in the interview data to support a direct link between word meaning as a mediator of thought and language on the one hand, and culture and ethnic origin on the other hand. This theme requires much further investigation.

Level of Understanding

The second theme to pursue with respect to cognition was level of understanding. This theme was informed by the suggestion that one's word meanings go through stages or evolve, and that mathematical understanding also evolves or progresses through levels.

There was some indication that the meanings the students were associating with words or symbols used in mathematics were going though levels or evolving. For example, Dorothy thought that 'subtraction' meant that you always got a smaller result. This is shown in her answer to Test Question 5 (ii). The question was if n-246=762, then n-247=...:

Excerpt 14:
Dorothy: You're subtracting this, but it went up!

Unfortunately for Dorothy, she was confusing 247 with the result of the operation; her comment referred to 246 being replaced by 247, a larger number. The result was in fact a smaller number, 761. As well, it is worth noting that Dorothy's understanding of subtraction may not have included the fact that the result of a subtraction *can* be a larger number than what you started with (for example, 9 - -2 = 11).

Jacob's understandings of some concepts also appeared to be incomplete. At one point Jacob was associating brackets with multiplication only, and not with other concepts such as order of operations. This is illustrated in his responses to Test Question 13, which involved simplifying algebraic expressions.

Excerpt 15: (Simplify (a+b)+a)
Mark: How about this one, when there's brackets?
Jacob: Oh, that's multiply.

Excerpt 16: (Simplify (a-b)+b)
Mark: What does -b+b give you?
Jacob: Negative and positive equals minus.

Jacob's response in Excerpt 16 suggests he is applying the multiplication rule of a positive number multiplied with a negative number, instead of realizing he actually was dealing with an addition problem. Also his understanding of the use of a letter as a specific unknown was unclear, as is seen in his responses to Test Questions 5 (iii) and 6 (i). Jacob correctly answered 5 (iii), that is, if e+f=8, then e+f+g=8+g. His reason is given in the following excerpt:

Excerpt 17:
Mark: Why 8+g?
Jacob: Because g doesn't have any value.

Then for the very next question, 6 (i), what can you say about a if a+5=8, Jacob replied with the same response, but this time incorrectly:

Excerpt 18:
Jacob: 'a' doesn't have any value.

It took a little prompting to get him to realize that 'a' was in fact equal to 3, suggesting that he had attained only a partial understanding of the various uses of letters.

Jane's understanding of multiplication seemed weak when it came to multiplying an expression by a number. This came to light in her response to Test Question 4 (v), which asked her to multiply n+5 by 4. Her initial answer was 4n+5, which was incorrect as she had only considered the first term 'n' in n+5. It took some prompting for her to realize that the entire expression n+5 was being multiplied by 4, and hence the answer should have been 4n+20.

With respect to the students' answers to the Chelsea Test questions, each demonstrated a different level of understanding - or degree of mastery - of algebra as generalised arithmetic. Also, due to the positive results produced by the students when I provided assistance, the second interviews suggest that the students were appropriately within their zone of proximal development

(Vygotsky, 1934/1986). In other words, grade 9 algebra was within the limits of their abilities. As noted in Chapter Two, only Level 3 and 4 questions were seen to truly deal with algebra. In addition, as noted in Chapter Three, of the ten codes developed to score the students' results, seven (Codes 3 to 9) refer to the process of attaining an incorrect answer. The discussion of each student's results begins with Jacob.

Jacob needed a fair amount of assistance with interpreting the test questions. Several of his initial attempts at answering them resulted in incorrect answers that were rated as premature closure (Code 8), letter not used (Code 7), or none of the above (Code 9). After prompting he was able to answer more questions, and he ended up correctly answering all Level One and Two questions, four of five Level Three questions, but only one of five Level Four questions. These results gave an overall impression of Jacob as having begun to understand the use of letters as specific unknowns but having not yet attained a level of understanding where he could treat letters as generalised numbers or as variables.

Dorothy had several incorrect answers before being assisted by me, indicating something was amiss with her understanding of algebra as generalised arithmetic. She too responded positively to prompting, resulting in an improvement in her test score. Dorothy completed the second interview with all Level One and Two questions correctly answered, but only two of five Level Three and none of four Level Four questions answered correctly. The codes assigned to her incorrect answers were for letter not used (Code 7), premature closure (Code 8), and none of the above (Code 9). Because of these process problems and only two correct Level Three or Four answers, I believe Dorothy was only just beginning to truly understand algebra as generalised arithmetic.

Jane had the best results, even without prompting. It should be noted that she was also the only one of the four students enrolled in a mathematics course at the time of the interviews; the other three students had probably not done any formal mathematics for four months. Working alone she correctly answered all of the Level One and Two questions, and four of five Level Three questions. Her errors were confined to premature closure (Code 8) and two ambiguous answers (Code 2). After prompting, she answered all Level Three questions and two of five Level Four questions correctly. Based on these results Jane had reached a high level of understanding of algebra as generalised arithmetic.

Kim had the most difficulty with the test. In fact, with several questions I stopped prompting her as it became clear she had no notion of how to solve them. She ended up correctly answering all Level One questions, two of three

Level Two questions, and no Level Three or Four questions. Her errors were coded as letter not used (Code 7), premature closure (Code 8), and none of the above (Code 9). In fact, nine of her answers were coded as none of the above; she did not demonstrate a strong grasp of most of the material. This result was despite the fact that Kim had completed the grade 9 mathematics course, and most of the questions easily fell within the Ministry of Education and Training's (1995) algebra and patterning outcomes as stated in The common curriculum: Provincial standards, Mathematics, Grades 1-9. Thus it appears that Kim's understanding of algebra as generalised arithmetic was non-existent, and that her level of understanding of the mathematics on which generalised arithmetic is built - such as dealing with some ambiguity within a problem - was not fully developed.

Consequently there was enough data from the transcripts to suggest that word meaning and mathematical understanding do move through levels. Whether or not those levels experienced particular cultural or ethnic effects requires more investigation, as the present research did not note any.

Motivation and Attitude

The third theme explored dealing with cognition, motivation and attitude, was particularly rewarding to explore. This was the one theme about which all the students had plenty to say. Because motivation and attitude are linked - the former being the reason or root of the latter - and also very individual or personal, the data are presented organized by student, firstly describing his or her attitudes towards mathematics, which were easier to determine than the motivators, then suggesting what could have motivated each student's attitudes.

Two of the students expressed a dislike for mathematics. Dorothy said it was easier when she studied mathematics in the refugee camp in Turkey because it was in her own language, but even then she did not like it. From our conversations the main motive for this attitude appeared to be her stated aversion to too much work.

Also, Dorothy identified strongly with the Kurdish people. This gave me the impression that national identity was a motivator. A sense of this comes from the following excerpt:

Excerpt 19:

Mark: When you were back in Kurdistan ... or in Turkey, and you were using these different numbers like you showed me, like when people went to market and so on, how did they pay for stuff? ... When you went shopping how did people calculate the money and stuff? Like, let's say, you know, here you go to a Loblaws and you buy bread. And you give them a $5 bill and they punch it into a machine and it tells you how much change to give you. How did they calculate things like that? Do you know?

Dorothy: They're pretty smart. I'm not saying you guys are stupid or anything. I'm just saying ...

My sense at the time she gave that response was that she was inferring that '*we* didn't need all those machines to do the work for us.' She seemed to be very proud of her 'Kurdishness.'

With respect to family as a motivator, Dorothy's father was a soldier who reportedly went to school for forty days to learn to read and write, and her mother never attended school. Dorothy did get help at home with her homework, both from her parents and from family friends who were working as teachers at the refugee camp. So perhaps there was a sense of the value of an education within the family, despite the parents' lack of an education.

Kim was the other student who disliked mathematics. This might be in part due to having moved frequently while she was in Asia, and consequently did not attend classes and fell behind. Kim frequently described herself as 'stupid.' Being hit for not being able to answer mathematics questions would also have helped build a negative attitude. Perhaps, too, the rebelliousness she expressed combined with her father's emphasis on the importance of mathematics was a factor. An indication of her attitude is seen in Excerpt 20.

Excerpt 20:

Mark: Is there anything you like about mathematics?

Kim: I don't like math at all, you know. My Dad tried to make me like it, but I said, "It's too hard." If it was easy I would love it, but there's a lot of people who like it because they're so (unclear) and they're so genius. Oh, one thing I like about it is plus. That's, like, the only easy one! And take away, well, not that much. And I hate dividing, it's so hard. Because you know you have a lot, like, 52 and there's tons of number at the back and you have to use these

> (unclear), I'm like ... I have no clue! I'm not even there yet. Some teacher just teach me those and I have no clue what to use with it. And, like, I get also confused and my head doesn't work and I'm thinking, "I can't do this, I can't do this," and afterwards I can't do it. And I can (unclear) do it!

That being said, during our discussions Kim did not strike me as 'stupid.' She came across as having a non-serious, playful personality, something she affirmed by saying that when she was a student in Asia all she was interested in was playing.

If fear of punishment serves as a motivator, then Kim should have been highly motivated. She reported being physically punished for making mistakes, both by her parents when helping her with homework and by her teachers. She spoke of helpful and unhelpful teachers, and described her favourite teacher in Asia as one who knew of her troubled background and did not hit her too much. Kim also indicated that impressing a teacher was a motivator for her:

Excerpt 21:
Kim: I had ESL English and the teacher, like, I don't get that much English. And then she gave me homework every night. And all the homework I always did it. And every day she was always see me doing homework. Soon I get more good and gooder at this. Cause I do homework every day. And then she gave me a math, like, grade 4 math. Grade 4, 5, 6. For three month I'd be finished the whole page cause I did every night at home and I did more other done and more. So she would be proud of me. That's what I always do. I just want the teacher to be proud. I did, like, for three month I finish one whole book and she was so proud me because I finish it and everybody hadn't catch up. And they usually, like, a year and they'd be done. So that kinda helped me, though, because I

> saw every number of math and a little bit, that time I did math.

Thus, for a while at least, Kim had a positive attitude towards doing school work as she was motivated by her teacher. With respect to Vygotsky's theories, this could have been a time when her cognitive structures underwent significant development.

But it was Kim's relationships with her parents that I felt acted as her key motivator. Kim described herself as being switched as a child, and did not know much about her mother. She seemed to be rebelling against the woman she was calling her mother, which could be a significant but not necessarily positive motivator. Her father was a boxer who later became a teacher. Kim spoke more kindly of him than of her mother, but there was still a sense of rebelliousness. She described her parents as being too protective, and compared herself unfavourably to her elder brother. She also did not appear to like her parents expectations for her, which included a cultural link as they wanted her to return to Vietnam to choose a husband. There was very little to suggest Kim's home environment motivated her in a positive way.

Jacob did not express an overall negative attitude towards mathematics, although he did express a dislike for fractions.

Excerpt 22:
Mark: Is there anything you don't like ... in mathematics classes?

Jacob: Fractions.
Mark: You don't like fractions? Why don't you like fractions?
Jacob: You gotta do many things. You multiply, you (unclear).

His comment could simply indicate an unwillingness to work, or perhaps something deeper - such as having difficulty dealing with tasks that require many operations or steps. In general, however, Jacob found the move to mathematics classes in Canada easy as he had seen most of the material before while attending school in Guatemala City. The only topic he said was new was algebra.

I sensed from our discussions that Jacob's motivation to learn may have come in part from pride in his school in Guatemala. There were two kinds of schools in Guatemala City, public schools and colleges. Jacob attended both, initially the more expensive college and then public school. His comments about them led to the point about pride, as is illustrated in Excerpts 23 and 24. In the first excerpt we are discussing what he found different about Canadian schools.

Excerpt 23:
Mark: Oh, you had to wear uniforms?
Jacob: Yeah, we had to buy uniforms.
Mark: In the college and the public school, or both?
Jacob: Yeah, the two of them.
Mark: And do you think that's good to wear a uniform? Or do like dressing in your ...
Jacob: Well, sometimes I like to use uniform.
Mark: Why is that?
Jacob: Cause you show your clothes. You don't have nothing to wear when (unclear) ... Or, when somebody sees you, they see that you are from that school. Cause you wear uniform.

In the second excerpt we are discussing whether there was an advantage to attending a particular type of school in Guatemala City.

Excerpt 24:

Mark:	If you go to the college, does it give you more ... I don't know ... chances to get jobs or things than public school? Do you know what I mean?
Jacob:	(unclear).
Mark:	Um ... The students who went all the way through in the college, when they finished did they go to university, did they get government jobs, did they have any ... was it better for them than the students in the public school?
Jacob:	Hmm. Sometimes it was because the teachers were more responsible.
Mark:	Oh. I see.
Jacob:	Yeah, cause in the schools we had ... they had a strike, sometimes we don't go.

My sense from these and similar comments was that he would have worked at his studies in order not to let the school down.

With respect to parents or teachers being motivators, Jacob had a favourite teacher in Guatemala whom he described as "so nice." He disliked teachers who had too much "self esteem." As for his parents, his father attended university to study law, his mother had a grade 6 education, and they would help him with his homework. Hence he had a teacher he may have wanted to please, other teachers he may have been less inclined to impress, parents who seemed to acknowledge the benefits of an education, and at least one parent who could model the benefits of schooling to him. All of these seemed to be positive motivators.

Jane's previous experiences of mathematics classes may have contributed to her attitude. She said that her favourite part of mathematics was algebra. Jane brought some of her Libyan text books to the first interview, and her grade 10 text contained material that is taught in Ontario's grade 12 advanced mathematics course. Some of the material introduced or expanded upon in Ontario's grade 9 I found in her grade 5 text. As is demonstrated in the following excerpt, Jane found the content of her mathematics class very familiar.

Excerpt 25:

Mark:	So this stuff, this ... You're saying, this grade 5. You did this in grade 5. So that would be five years' ago, you did that?
Jane:	Yeah.
Mark:	So, is there anything in grade 9 you're doing now that's new?
Jane:	No.
Mark:	You've seen it all before?
Jane:	The same. Everything.
Mark:	Wow. That's incredible ...

Hence it is possible that Jane's positive attitude may have stemmed from her previous exposure to - and depth of experience with - the material.

Jane reported that in Libya all girls went to school, and that she had female teachers at school. She did not understand the question when I asked about equal or similar opportunities for both boys and girls, but also never indicated that she thought girls were not treated equally. One could speculate that this meant she could have been motivated by a variety of role models. Jane had one teacher, a woman, whom she particularly liked. She also had teachers whom she thought were not very good, and was sometimes physically punished for making mistakes. The following excerpt illustrates these two last points.

Excerpt 26:

Jane: My Dad said to my ... said to Mrs. **** [the ESL teacher] the teacher in math not good. The teacher (unclear) science and just work work work you ...

Mark: These are the teachers in Libya?

Jane: Yeah.

Mark: So ... when you used this book, for example this book in grade ... this is interesting, I find this very interesting. When you used this book in grade 5, so let's say [points to an exercise in the Libyan text] ... okay ... When your teacher gave you this problem to do, how did your teacher know if your answer was correct or not? Like, when you do the answer, you say your father said it was very quick, how did your teacher check to see that you were doing the right thing?

Jane: We don't have that book. To check it, yeah. Sometimes they show you work, and they correct. Just the homework, homework they (unclear) like this when you get ... When your answer is wrong they get you a mark. [This may mean the student had a mark deducted.]

Mark: They hit you with a stick? [I have clearly misunderstood her answer.]

Jane: Sometimes, yeah. You don't understand, you have to ask. You have (unclear). I don't know.

Mark: But when you got it wrong, how did you learn to make it right? Like, if you know you are making a mistake, or if the teacher sees you are doing this wrong, how did the teacher correct? Like, how did you learn to do it right?

Jane: The teacher write it ... under your answer.

Mark: And you just sort of see what the teacher did ...

Jane: Just write it, don't (unclear).

This suggests that, as with Kim, fear of punishment could have been a motivator.

With respect to Jane's home environment, she reported that her parents began teaching her the English alphabet when she was six years' old. She also said that nobody at home helped her with homework, commenting that she was the eldest child. It was not clear what birth order had to do with it, but perhaps her culture valued individual achievement, or perhaps she was indicating that no one had been around when she needed help but that she would be able to help her younger siblings. Or perhaps in her culture only siblings helped each other; it may not have been a parental responsibility. In sum, with the exception of the emphasis to learn English, Jane did not indicate her home environment was a particularly strong motivator.

Overall, the interviews with the four students did reveal a variety of factors affecting each student's motivation and attitude. What did not appear in most cases was an identifiable cultural or ethnic root for the factors.

Concepts

The final theme explored regarding cognition was the development of concepts. Investigating this particular aspect of cognition theoretically required the students to have an awareness of their learning processes. These processes included generalization, identification of features of objects, comparison and differentiation, and synthesis. The results of probing the four students for indications of these processes occurring are now presented.

Three of the four students indicated how their concepts were, or were not, developing. As has been demonstrated with the analysis of the Chelsea Test results, Kim had the weakest conceptual understanding of algebra as generalised arithmetic. She attempted to describe why her understanding was so weak during the course of our discussions:

Excerpt 27:

Mark: What don't you like about algebra?

Kim: It confuse me. The letter ... alphabet, and the number, and this little x sign, and, like, I had no clue what to do. Once you get what you're doing, like, I get there, but if you go to the new one, like, the next assignment, but almost the same but you have to do something different, I get all confused. I had no clue what the teacher trying to tell me, so I'm like, "I don't know what (unclear) you're talking about!" So, that one is like the hardest one for me. I never get that one right.

Mark: Is it hard because the teacher's using words you don't understand?

Kim: To me, it's just the way the math is ... it's just this ... the number and the letter ... I just don't get it. It's just weird. I don't understand them. It's just so hard. I failed that one, you know, I failed those math. I just don't get them. It just makes me feel so hard. I just can't do it!

Part of what Kim said in Excerpt 27 indicates she had difficulty with the process of comparison and differentiation, when she spoke of understanding one problem and then getting confused by a similar problem. More specific indications of what was causing her confusion came from the second interview as she tried the problems from the Chelsea Test. Kim had not grasped the concept of a letter being used as an unknown or a variable. For example, with Test Question 3, which is larger, 2n or n+2, she responded:

Excerpt 28:

Kim: Same thing. 'n' has to be some number. If 'n' has no number, then n+2 is 2.

With Test Question 5 (i), if a+b=43 then a+b+2=..., she said:

Excerpt 29:

Kim: You have to give some number to get the right answer.

This response indicates that she felt both 'a' and 'b' had to have specific values, so she was not generalizing. Even with prompting she had problems, as is seen with Test Question 13 (iii), simplify (a+b)+a, where she appears to have difficulty synthesizing or putting together all the parts of the problem:

Excerpt 30:

Kim: a+b=b, plus 'a' is ... I'm stuck.
Mark: How many 'a's?
Kim: Two 'a's, and 'b', they're plussed, ab.
Mark: Can you add 'a's and 'b's together?
Kim: Yeah, you could. The second 'a' is like the first number and 'b' is the second number, and if a+b, so 4 plus one more, 5.
Mark: ... adding apples and oranges ...
Kim: Can't add different things. Oh I know, a+a=a.

Kim's last remark in Excerpt 30 does suggest she had some awareness of the process of generalization. However, after several responses like this one I cut her second interview short, as she was not having much success. I decided that, in terms of exploring Kim's mathematics learning experiences, there was nothing to gain from continuing.

Dorothy had difficulty adapting her understanding of a mathematical operation when both numbers and unknowns were involved. In particular, she was not acknowledging the operation signs included within the questions. For instance, with Test Question 4 (ii), 4 added onto n+5:

Excerpt 31:
Mark: What's n+5+4?
Dorothy: n5. Oh, 9n ...
Mark: So what happened to the plus sign?
Dorothy: What do you mean?
Mark: You said this ... You're telling me that 4+5+n, you're saying that's the same as 9+n, this is 9, and you got 9n. So I'm asking what happened to the plus sign?
Dorothy: You take that out.

Similarly, with Test Question 13 (v), simplify (a-b)+b:

Excerpt 32:
Dorothy: It would be a-2b.
Mark: Where's the 2b coming from?
Dorothy: Because there's one (pointing to the first 'b') and two (pointing to the second 'b').

Excerpt 31 indicates that Dorothy was generalizing the rules of addition incorrectly. Excerpt 32 suggests that she had not identified a feature of the question, which was that one b was being subtracted but the other b was being added. Dorothy's concept of simplifying expressions seemed to not allow for the significance of the operation signs for addition and subtraction.

Similar to Kim, Dorothy also had difficulty grasping the various uses of letters in generalised arithmetic. In Test Question 5 (iii), if e+f=8 then e+f+g= ..., she wanted the letter to be assigned a value:

Excerpt 33:
Dorothy: What's 'g' anyways?
Mark: 'g' can be anything ...

In Test Question 6 (ii), what can you say about 'b' if b+2=2b, her response was just the opposite. She did not expect the letter to have any value:

Excerpt 34:
Dorothy: 'b' doesn't do anything. It's just a number ... it just a letter.

Surprisingly, later she correctly answered a question involving three unknowns, which could imply that she was clarifying her concepts as we worked through the test questions. This was with Test Question 14, what can you say about 'r' if r=s+t and r+s+t=30:

Excerpt 35:
Dorothy: 15.
Mark: How did you get 15?
Dorothy: Because I divide 15, I mean 30 into 2. I get 15. Because if 'r' is that (points to r=s+t) and there's another 'r', so I get two of these and two of these.

Notice that Excerpts 33 and 35 could indicate that Dorothy was getting better at the processes of identifying features of the problems and of comparing and differentiating, which may have been the cause of the clarified concept. Connecting with another cognition theme, the preceding discussion could also imply Dorothy was reaching a new level of understanding.

Jacob also indicated that his concepts of generalised arithmetic were not fully developed. For example, with Test Question 3, which is larger, 2n or n+2, he answered 2n because:

Excerpt 36:
Jacob: n+2, you can't do nothing.

He was aware that there was a symbolic difference between

2n and n+2, so he has identified features of the problem, and he knew that 2n meant that something was being done with the 2 and the 'n', but his concept of letters as unknowns did not allow him to recognize that he could in fact add 2 to 'n' if 'n' were to take on some value. Thus he appeared to be held back by the process of synthesis.

All four of the students stumbled with Test Question 22, the word problem involving blue and red pencils. Their difficulty lay in first of all understanding the problem, and then changing the sentences into one or more equations. The question itself was complex, and would have required all the processes of generalization, identification of features of objects, comparison and differentiation, and synthesis to successfully answer. None of the students completed the problem, but all seemed to be trying to find a numerical solution. This was not possible with the information provided in the problem.

The exploration of this theme of concepts has found indications of support for Sierpinska's (1993) theory that the development of concepts requires the processes of generalization, identification of features of objects, comparison and differentiation, and synthesis. What remains unclear is how each student was aware of their use of those processes, so a distinctive cultural or ethnic aspect to the theory was not perceived.

Summary

The explorations with the themes pertaining to cognition have been fruitful in the sense of supporting most of the theories that motivated using the themes in the first place. Level of understanding, motivation and attitude, and concepts all appear to have played an important part in the mathematics learning experiences of the four students. However, with respect to the fact that the students were also immigrants, a distinct cultural or ethnic effect on those themes was absent.

I find this a most striking and unexpected observation. The reviewed literature strongly suggested cognition as being important for the learning of mathematics in general. The links connecting cognition to culture and ethnic origin, both explicitly in the literature on cognition and implicitly from my reflections on the literature on language and culture led me to anticipate a distinctive cultural or ethnic flavour to the results of analyzing the cognition constructs. But three of the four students appeared to possess a way of thinking about and working with algebra that fits within the range that would be expected of any grade 9 class, and the fourth - Jane - had a tremendous advantage not from her culture or language, but from her exposure to the material at a much earlier age than is the norm in Ontario. If cognition is knowing, as was suggested in Chapter Two, what these four students 'knew' - at least as far as grade 9 algebra was concerned - was on a par with their Ontario born and educated peers. The data presented under the heading of cognition could just as easily have come from students born and raised in Ontario.

Overall my analysis supports the importance of seven of the initial themes with regards to the mathematics learning experiences of the four immigrant students: proficiency in the working language, words or symbols used in a language, multiple systems within a single culture, cultural inhibitors, level of understanding, motivation and attitude, concepts. I did not see support for the four other initial themes: bilingualism, culturally different curriculum, minorities, word meaning. All of these themes require further exploration, the first group because of the importance my analysis of the students' experiences attached to them, and the second group because of the importance the reviewed literature attached to them.

What was not expected, and which also requires further exploration, was the lack of cultural or ethnic aspects to what I observed of the 'supported' group of themes. I repeatedly noted that the data I was analyzing could have come from a student born and raised in Ontario.

This brings to an end the presentation of the results of my research. However, these results lead to further speculation. For example, why was it that the four students were not dramatically different from their peers, as far as mathematics learning experiences are concerned? Could it be that D'Ambrosio's (1990) cultural genocide has produced a global mathematics replacing ethnomathematics? In the final chapter questions such as these are considered as my results are summarized and discussed, and conclusions drawn. Also included is discussion of limitations and suggestions for further research.

CHAPTER FIVE

Conclusions

In this study a conceptual framework was developed to facilitate research regarding cultural and ethnic factors which could affect the learning of mathematics in an Ontario classroom. The framework was based on ideas and theories gathered from a review of selected literature. The framework was used to develop a set of research questions which were then given to four grade 9 students at a London, Ontario secondary school during two sets of interviews. Interviewing the four students allowed me to seek possible support for ideas from the literature by reflecting upon the mathematics learning experiences of these students. In this chapter, a discussion of the findings of this research is presented. This chapter also contains discussion of the limitations of this study, and ends with suggestions for further research.

Discussion of Findings

The intent of this research was to explore how culture and ethnic origin had an effect on a student's learning and understanding of mathematics. The first finding of this exploration was that there existed a huge body of literature pertaining to the subject. Some of this literature dealt with the learning and understanding of mathematics, some investigated cultural or ethnic issues, and a smaller portion addressed both. The mere existence of this body of literature speaks of the importance the research community and its backers attach to the subjects of culture, language, and the learning of mathematics. In addition, given that the literature came from countries such as Britain, the United States, South Africa, Botswana, Papua New Guinea, and Brazil, it seemed evident that these subjects are of global concern.

It was also evident that some caution had to be taken while researching connections between culture, language and the learning of mathematics, for, as Secada (1992) warned, the researchers had to take care against creating or reinforcing group stereotypes. This caution was reinforced by my research. Some of the preliminary reports I read which attempted to connect culture or language to school mathematics performance could be interpreted as creating expectations for the mathematics performance of particular groups of students. Some of the students participating in my research belonged to some of those identified groups. But in terms of the mathematics the students did during the second interviews their performance did not correspond to what the statistical reports could lead me to expect. Furthermore, I did not detect any significant aspect of the students' Ontario mathematics learning experiences that would have been unanticipated from an Ontario born and raised student.

Once my exploration of the literature was underway, the next finding was that first two, and finally three, headings emerged as being important for an immigrant student's mathematics learning experiences. These headings were language, culture and cognition. It was cognition which was the last heading to emerge. It was also cognition that, based solely on the literature, I believed would be the most important factor in the immigrant student's learning of mathematics. In particular, Vygotsky's theory of cognition provided a logical link between a student's culture and language on one hand and a student's learning of mathematics on the other hand. A student's culture could serve as an external mediator for thought - including thought on mathematics - and the word meanings assigned by the student's language could act as an internal mediator. My research was not able to properly investigate the cultural or ethnic effects of word meaning. Indeed one of the key observations of my research was of the lack of evidence of cultural or ethnic effects, which raises the question of just how important different cultures and languages are as mediators for thought. However, my research did support the importance of cognition itself to the students' learning of mathematics, reinforcing in my mind the impressions created by the literature.

The third major finding of my explorations was the identification of significant themes within the three headings of language, culture and cognition. For language, the themes were proficiency in the working language, words or symbols used by a language, and bilingualism. For culture they were culturally different curriculum, multiple systems within a single culture, cultural inhibitors, and majority versus minority culture. And for cognition the themes were word meaning, level of understanding, motivation and attitude, and concepts. These themes informed the development of a conceptual framework which was used to explore the mathematics learning experiences of four immigrant students. The findings of that particular exploration are provided in Chapter Four. Support was seen for the themes of proficiency in the working language, words or symbols used in a language, multiple systems within a single culture, cultural inhibitors, level of understanding, motivation and attitude, and concepts. I did not see support for the themes of bilingualism, culturally different curriculum, minorities, and word meaning. Overall it seemed that the students' transition to an Ontario mathematics classroom was fairly easy. However, some critical findings of the exploration of the themes need to be discussed here.

In the review of the literature on bilingualism, Saxe (1988) was reported to conclude that growing up bilingual positively influences some aspects of a student's cognition about the arbitrariness of numerical conventions. Clarkson

(1992) found that students competent in both English and their mother tongue had an advantage in mathematics achievement over students competent in just one language, even when the monolingual students had significant material advantages at their schools. These two statements are significant in that they probably run counter to some people's predictions. As well they reinforce the suggested relationship between language and cognition. There are several possible explanations as to why my research did not find support for these statements. All have to do with factors that could have nullified the effects described by Saxe and Clarkson. The first explanation is to question what bilingualism truly means. If a student has to be very proficient in English and another tongue, then it may be significant that the most proficient English speaking student participating in my research had lived in Canada for the majority of her life. Consequently many of her mathematics cognitive structures would have been formed by, and dependent upon, English.

Other explanations include the fact that three of the students used the same numerals and mathematical symbols in their home countries and two of the more recent arrivals in the group previously had been exposed to the material they were taught in Ontario. Not only do these explanations potentially nullify the effects of bilingualism, they also highlight D'Ambrosio's (1990) discussion of the 'civilizing mission.' The fact that three of the four students used the same numerals and mathematical symbols in their home countries, and also reported learning similar material to what they were exposed to in their Canadian classrooms, lends support to D'Ambrosio's notion of 'cultural genocide.' The 'new society,' at least as far as mathematics is concerned, has reached into south-east Asia, Guatemala, and Libya. Kim, Jane and Jacob had no sense of a mathematical heritage other than that of their eurocentric schools.

Closely related to the lack of a sense of a mathematical heritage was the fact that the content of mathematics classes was apparently similar in Libya, south-east Asia, Guatemala, Kurdistan and Canada. This fact may account for my research finding support for the cognitive themes of levels of understanding and concepts, but not finding support for a cultural or ethnic effect on those themes. The same reason may also account for the lack of support for the theory of the negative effects of a culturally different curriculum. The implication is that there may truly be a global mathematics 'culture,' at least in terms of curriculum. This result of my research was an interesting one in light of the statements in Chapter One against a uniform treatment of all students; it means that these four students would not have mathematical needs noticeably different from their Canadian peers.

In addition, all four students came from cultures and societies in which writing was common. Even in Jane's Libyan texts, which were written from right to left in Arabic, the numbers were perfectly understandable to me. Perhaps this fact suggests another link between language and cognition: is it possible that languages which include writing encourage the development of similar cognitive structures?

Another finding of the literature review was the theme of the effects of being a member of a minority culture. Dorothy's time as a refugee in Turkey may qualify her as once being a member of a minority. As detailed in Chapter Four, Dorothy's refugee experience may have strengthened her national identity but otherwise did not appear to have affected her mathematics learning experiences. This observation is an interesting one to reflect upon, because it does not seem unreasonable for someone with such a strong national identity to be motivated to perform well at school. Her pride in being Kurdish could have driven her to want to succeed academically, perhaps in order to later be able to do something positive for her people. This consequence did not appear to have happened. A counter hypothesis, for example, is that the exact opposite effect occurred; Dorothy's national identity could have caused her to reject the ways of the foreign society she found herself in. Recall for example that in Excerpt 19, when I asked

Dorothy how calculations were done when her family went shopping, Dorothy's first response was that her people are pretty smart. One way to interpret this response was that she was defensive about the different ways her people did things in her home country. In Excerpt 8 Dorothy spoke of how shocked she was at the manner of dress and use of makeup by people in Canada. It is conceivable that these excerpts suggest a dislike for Canadian ways of doing things, which could also be manifested in a dislike of school - including mathematics class.

As has been noted, the exploration of the literature created a sense of the importance of the theme of word meaning, but the explorations with the students yielded nothing explicit showing how word meaning mediated thought and language for the four students. For example, when a student was having difficulty with one of the Chelsea test questions, it was unclear from the discussions whether the reason was due simply to having difficulty with a particular English word or a concept expressed in English, or if there was a deeper issue of a conflict with a pre-existent cognitive structure. Perhaps the lack of evidence of a cultural or ethnic connection to word meaning was due to the fact that for Kim, Dorothy or Jacob the mathematics material we dealt with during the interviews had been learned by them after they had moved to Canada - and thus was learned in, and the cognitive structure formed by, English. If that was the case, then their structures could be similar to mine as I too learned generalised arithmetic in English, making identifying an unusual characteristic very unlikely. In Jane's case the cognitive structures she was using may have been informed by the language she spoke in Libya, but her long familiarity with the mathematics material combined with her low English proficiency could have effectively prevented her from communicating to me a sense of those structures.

The findings of the exploration into the theme of motivation and attitude are worthy of some discussion, as three items arose which appeared to be linked to culture and ethnic origin. One was the use of physical punishment in some of the schools outside of Canada attended by the students. As a motivator this could have had a negative affect on attitude towards school and learning, and also could have motivated the development of instrumental instead of relational understanding. The second item was pride as a strong culturally-based motivator. This came up both with Jacob's explicit statements about his school and uniform, and with Dorothy's apparent national identity. In Jacob's case it may have helped with his apparently positive attitude towards school. However, the cultural or ethnic significance of these two items on the mathematics learning experiences of immigrant students is ultimately questionable, as the end results of pride, a dislike of mathematics, or instrumental understanding of mathematics are also found amongst Ontario born and raised students.

The third cultural or ethnic item identified for the theme of motivation and attitude has to do with the early exposure of many of the students to the material they later saw in Canada. This could have placed them at an advantage, as the cognitive structures needed in order to work with the material may have already been established. It does seem conceivable that such an advantage could remain for the rest of the student's years at school for, while the student's peers were working on developing the structures he or she already had in place, the student would be free to develop new structures.

Having made that statement, it is interesting to consider the effects of rote learning, which may have been practised in the countries which exposed students to mathematical concepts at an earlier age than their Ontario peers. The two students who appeared to have experienced rote learning had vastly different performances on the Chelsea Test questions. Kim was one, and Jane was the other. Strictly in terms of outcome, Kim had the lowest achievement of the four and Jane had the highest. Kim had difficulty generalizing the basic operations of arithmetic, operations she may have begun to learn by rote before moving to Canada. On the other hand, Jane definitely knew her mathematical rules for generalised arithmetic, but it was unclear from our discussions whether she understood the rules or was merely applying them in situations she recognized from previous work. It was difficult to determine what their understandings really were, but again these cognitive observations are likely not a cultural nor ethnic origin issue.

Thus the findings of the literature review portion of my explorations are largely unsupported by the interview analysis portion of my explorations, possibly due to the effects of what D'Ambrosio called 'cultural genocide' and the 'civilizing mission.' In addition, if one accepts the existence of a global mathematics curriculum 'culture,' then one would not expect to find a difference between an immigrant student and a student born and raised in Ontario. They would belong to the same culture. The lack of support should not weaken the importance of the findings from the literature in general, but does question the importance of many of those findings as they pertain to ethnic and cultural effects on immigrant students' experiences of learning mathematics.

Mathematical Results from the Chelsea Test Questions

Some discussion is appropriate on the Chelsea Test's definition of levels and how the students performed with respect to those levels. Küchemann (1981) defined both levels of understanding and categories of interpreting and using letters when describing how to analyze the test results. The set of questions asked of each student in the second interview covered all six categories and all four of Küchemann's levels. The students' performance with respect to the four levels was recorded, following the test's recommended procedure.

Each Chelsea Test question was assigned a category. However, Küchemann also stated that only three of the six categories - letter used as a specific unknown, letter used as a generalised number, letter used as a variable - indicated the students had attained some mastery of generalised arithmetic. In addition, Küchemann assigned to some of the questions a ranking of levels from 1 to 4. Of those levels, only Levels 3 and 4 corresponded to generalised arithmetic. This resulted in what could appear to be a conflict between generalised arithmetic categories and levels. Some of the questions used were in a non-generalised arithmetic category, for example, letter used as an object, but have a generalised arithmetic level, for example Level 4. The distinction lies with the fact that some generalised arithmetic *questions* could be answered by non-generalised arithmetic *methods* such as trial and error, and vice versa.

That being said, the two students who performed best in terms of levels - Jane and Jacob - were also the only ones to demonstrate higher order thinking with generalised arithmetic in terms of categories. Jane correctly answered three of five of these questions correctly, and Jacob answered two of five correctly. They could be said to have demonstrated a partial mastery of generalised arithmetic, while Dorothy and Kim demonstrated little or no mastery. While not a thorough investigation of each student's understanding of algebra as generalised arithmetic, this data does suggest each student's cognitive structures with respect to generalised arithmetic were at a different levels of development. Connecting with the themes drawn from the reviewed literature, this suggestion indicates that Jane and Jacob had developed some skill in their use of generalization, identification of features of objects, comparison and differentiation, and synthesis - the processes Sierpinska (1993) said were needed for the development of concepts - and therefore had moved to higher levels of understanding of generalised arithmetic.

Implications for Schools

The findings of this study produce three points of discussion for teachers of mathematics in Ontario. These points have to do with social factors versus cultural and ethnic factors, the lack of differences noted when comparing the students participating in this research to students born and raised in Ontario, the effects of prior knowledge, and the implications of language proficiency.

First of all, the reviewed literature suggested that culture and ethnic origin might have implications for the learning of mathematics. Teachers should note, however, that for the four students participating in my research, culture and ethnic origin did not appear to be significant determinants of mathematics performance. Culture and ethnic origin seemed to have some effects on the four students, but it may be that social factors had more effect. Of the four students, the ones with the weakest performance on the mathematics portion of the interviews were also those with the most disrupted social backgrounds. Those disruptions included war, refugee experience, moving from country to country, and family conflicts. The students with the strongest performance on the mathematics portion of the interviews appeared to have had relatively stable lives. Thus teachers should be aware that literature produced by ethnocultural research may not be directly applicable to the learning of an immigrant student, a point emphasized by the remarkable lack of differences observed between the students participating in the research and students born and raised in Ontario.

The second point derived from this research is that some attempt be made to determine what a new student's prior knowledge of mathematics is before placing that student in a specific mathematics class. This study found that some of the students were ahead of their Ontario peers in terms of the mathematics content they had studied. In Jacob's case, as he stated that he found fractions difficult and had not yet been exposed to algebra when he arrived in the country, placing him in a grade level which had not yet begun algebra and which allowed him more practise with fractions seemed an appropriate decision. But in Jane's case, based on the mathematics test she showed me during our first interview, on her performance on the Chelsea questions during the second interview, and on her Libyan mathematics text book which showed me that she had begun working on material at least two grade levels beyond the grade 9 material, I wondered if she could have been more profitably placed at a higher grade level for mathematics. In Vygotskian terms, I felt that the limits of her zone of proximal development for mathematics extended well beyond what she would be exposed to in grade 9. Her major difficulty seemed to be with the language of communication and not with the course content, which leads me to my final implication.

My observations of Jane raise the question of how to address an immigrant student's language proficiency. Language proficiency was clearly a factor affecting the students' ability to participate in mathematics class. Referring back to the reviewed literature, Spanos, Rhodes, Dale, and Crandall (1988) said that a minimum level of English proficiency is required for a student to function effectively in cognitively demanding tasks, and can take five to seven years to develop in a student's second language. They also discussed the mathematics register, and suggested that a language approach to mathematics education is required in order to develop this register. The importance of English proficiency for the learning of mathematics plus Spanos et al's requirement of a minimum level of proficiency suggest that educators should consider attempting to determine the English proficiency of a newly-arrived student before deciding to place him or her in a mathematics class. An associated implication is that mathematics vocabulary, indeed vocabulary specific to any secondary school subject, should be an intrinsic part of an ESL curriculum.

Limitations

Limitations of this research likely relate to time, the framing of the initial interview questions, and the eurocentric nature of one of the data gathering tools.

Time could have been a limitation of this study. After negotiation with the school and approval by the Ethical Review Committee, two interviews were conducted with each student, with a maximum of forty five minutes allowed for each interview. The time allowed was enough for a thorough discussion of all the prepared research questions. The four students all volunteered their time, were very accommodating, and did not give the appearance of being disturbed by the proceedings. However, if the students were not familiar with being interviewed, it is possible that they could have suffered from some anxiety during the interviews. Hence over a longer term the student may have become more relaxed and given different or more comprehensive answers.

Entering the interviews with a pre-conceived set of questions was a possible limitation. Although the questions did allow for all the factors and themes of my research to be explored, they also limited the use of probing when a student's answer diverged from the theme under exploration. A more open-ended set of questions would allow for more such probing.

Finally, another limitation of this research may have been the use of a eurocentric diagnostic tool - the Chelsea Diagnostic Test - to help look for cultural or ethnic factors affecting the learning of mathematics. Although the focus of my research is on the process of learning mathematics, it should be noted that the content of the test is very similar to what the students would have been exposed to in grade 9 mathematics class. The ways of responding anticipated by the test may have been governed by a particular eurocentric cognitive structure. Consequently it is possible that the test interfered with discovering some of the cultural or ethnic factors. Furthermore, the Chelsea test questions used focused on generalised arithmetic, which is based on what could be familiar arithmetic rules. It is possible that another test addressing a different topic of mathematics could have allowed exploration of the students' cognitive workings not permitted by generalised arithmetic.

Although not a limitation of the study per se, a different group of four may have answered the questions differently, perhaps leading to explorations of cultural or ethnic factors not possible with the participants of this study.

Suggestions for Further Research

In the course of preparing for, implementing, and writing up this study, a number of interesting questions arose. These questions suggest possible areas of further research related to the topic of culture and ethnic origin as it affects the learning of mathematics. Four suggestions are given.

First of all, the interview aspect of this study dealt exclusively with the perspectives of the four immigrant students. But there are other participants in the classrooms who may also affect the learning of these students. There are the other students in the classes, who will contribute in some way to the learning environment. There is also the teacher, who is the one facilitating the learning. It would be interesting to investigate what the preconceptions and perceptions of these two groups are as they relate to students joining their classes from other countries. Do any stereotypes come in to play? Is some effort made to educate the teacher and students on the background of the new arrival? What efforts are made to discover how the newly arrived student learns?

Secondly, this research was based in part on the theory that a student born and partially educated in another country could have developed a cognitive structure that affected his or her learning of mathematics. Investigating cognition thoroughly requires time, especially when one considers that the development of cognitive structure takes place over time. When collecting data from students, as they may not be fully aware of how their own structures are developing, other ways of gathering data besides interviews would be required. With time, students could be observed while in the process of learning. They could keep journals, providing data on their own thoughts. And they could be interviewed repeatedly over time, for example at the beginning, middle and end of their algebra unit, so that further data on how their cognitive development was progressing would be gathered. In addition, algebra need not be the sole focus, and topics such as geometry, measurement, estimation, and statistics could be considered. These other topics have the potential of exposing other cognitive structures not addressed by algebra.

Hence researchers should follow a recently-arrived student through a mathematics course, or a unit within a course, observing and recording the student's progress as well as interviewing the student throughout the time period. That process should reveal more comprehensive data on cognitive systems than this research did. It could include diagrams, charts, written samples of student work, transcripts, excerpts, observations of how a particular type of problem was attempted, and so on.

Thirdly, the analysis performed on the interviews conducted during this research did not find much to support or refute theories based on culture or ethnic origin. However the analysis did suggest that social conditions, although not explicitly looked for, had an effect on learning. As noted in Chapter Three, I decided to add 'Stability' as an unexpected construct or theme emerging from my research. This decision was based upon attempting to connect the students' life stories with their apparent mathematics performance. Kim had the poorest mathematics performance of the four students, and perhaps the least positive attitude towards school. She was also the student who had had the most disrupted life, moving throughout south-east Asia and perhaps losing her original parents before arriving in Canada. Dorothy, too, had some disrupted years, having spent four years in a refugee camp and one year in Vancouver before settling in London. Her mathematics performance and attitude were slightly better than Kim's. Jane and Jacob had both experienced what appeared to be relatively stable home and school lives, and seemed to be adapting well to attending school in London. This was despite their much shorter stays in Canada, as compared to Kim and Dorothy.

The emergence of stability as an unexpected theme or construct raises the question of its importance to the mathematics learning experiences of an immigrant student relative to the themes already discussed in the reviewed literature. This question merits further exploration.

Finally, there is a question of a philosophical nature. It is a disturbing question, because it presents a conflict between what it would be proper or moral for a teacher of mathematics to do, and what is practical to do. The question has to do with how teachers should treat 'other' mathematics. Thomas (1996) has called 'other' mathematics as described by D'Ambrosio and Ascher as 'proto-mathematics,' and cautioned against treating it as 'real' mathematics. Proto-mathematics includes culture-specific activities such as the mathematics used by adolescent street sellers in Brazil, by illiterate subsistence farmers in southern Africa, and by people anywhere who have not experienced formal eurocentric schooling but still practise some form of calculation. Thomas said that 'real' mathematics is the abstraction of what various cultures have developed with their proto-mathematics; it is the plant that has grown from the seed of proto-mathematics. Thomas also argued that 'real' mathematics is the truly global mathematics, as it was built on ancient Greek and East Indian concepts and has been practised and developed all around the world. In this he is supported by the fact that three of the four students participating in my research were learning 'real' mathematics in Guatemala, Libya, and south-east Asia.

That being said, the question remains of what to do with students arriving in Ontario classrooms with experience of 'other' mathematics. How can teachers acknowledge and give value to this experience while also ensuring that these same students learn the kind of mathematics required for active participation in Ontario's society? For, despite our stated political policy of multiculturalism, there is really only one kind of mathematics being practised in Ontario's businesses and industries, and that is the mathematics formally taught in the school system.

Conclusion

This research began with the question of how culture and ethnic origin had an effect on one's learning and understanding of mathematics. After developing a conceptual framework based on ideas selected from a review of relevant literature, my research explored this question with four grade 9 students, all of whom had been born and partially raised outside of Canada. This portion of my research focussed on questions of cognition, language, and culture by using the framework to analyze interviews with the four students.

The literature review portion of the research initially uncovered some quantitative studies and some discussion of ethnomathematics. Later the review identified the three headings of cognition, language and culture, and later still produced eleven themes which were theoretically important for an exploration into language, culture and the learning of mathematics. From the quantitative studies I discovered expectations for how particular ethnic or language groups would perform in a 'western' mathematics class. From the ethnomathematics literature I gained an understanding of just how vast are the activities practised worldwide that can be considered 'mathematics.' And from the three headings and eleven themes I learned just how complex and multi-faceted was the question I had chosen to explore.

The interview portion of my research was very helpful for indicating which parts of the literature review could truly be significant for the mathematics learning experiences of an immigrant student. First of all, I found that my preconceptions from the quantitative studies were wrong, endorsing Secada's (1992) caution about group stereotypes. The students helped me realize that, despite the variety of mathematics that has been identified worldwide, I should not expect that an immigrant student has been exposed to mathematics different from the kind practised in Ontario schools. And, although providing me with data that indicated support for many of the themes of language, culture and cognition in general, the students also helped me realize that for them culture and ethnic origin may not have played a major part in their mathematics learning experiences as immigrant students.

And so it is that the effects of culture and ethnic origin on the learning of mathematics need further exploration. The literature created a view that immigrant students' cultures, languages and cognitive structures will affect their mathematics learning experiences. But the four students interviewed during my research have challenged that view, suggesting that factors such as a common mathematics curriculum 'culture' and social stability may be equally important. Mathematics teachers working with immigrant students should therefore be somewhat cautious when applying views expressed in the ethnocultural literature to their classes.

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